Queen Mary
University of London

Main Examination period 2018

MTH5103: Complex Variables

Duration: 2 hours

Write your solutions in the space provided in this exam paper.

Student number: | | | | | | | | | |

Desk number: | | | |

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: M. Shamis

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This page is for marking purposes only:

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Question	Mark	Subpart Breakdown
1		
2		
3		
4		
5		
TOTAL :		

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Question 1. [20 marks]

(a) Compute the number

$$(i + i^2 + i^3 + \cdots + i^{101})^5.$$

Justify all of your steps. **[10]**

Write your solution to Question #1(a) below

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(b) Find all the solutions $z \in \mathbb{C}$ of the equation

$$z^7 = i.$$

Please write the answer either in Cartesian or in polar form. **[10]**

Write your solution to Question #1(b) below

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Additional space for Question 1

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Question 2. [20 marks]

(a) State the Ratio Test for complex series. **[10]**

Write your solution to Question #2(a) below

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(b) Using the Ratio Test, or otherwise, determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n.$$

[10]

Write your solution to Question #2(b) below

Additional space for Question 2

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Question 3. [20 marks]

(a) Find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$$

of

$$f(z) = \frac{2+3z}{z^2+z^4}$$

on the annulus $0 < |z| < 1$. **[10]**

Write your solution to Question #3(a) below

(b) Determine the residue of $f(z)$ from (a) at the point $z = 0$. **[10]**

Write your solution to Question #3(b) below

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Additional space for Question 3

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Question 4. [20 marks]

(a) Find all the branch points of

$$f(z) = (z+2)\mathrm{Ln}\left(\frac{z-2}{z+i}\right) + 5,$$

where $\mathrm{Ln}\, z$ denotes the complex natural logarithm. Justify your answer. **[10]**

Write your solution to Question #4(a) below

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(b) Find all singularities of the function

$$f(z) = z^6 \cosh\left(\frac{1}{z}\right),$$

and determine the nature of each of these singularities (e. g. removable singularity, pole, essential singularity). **[10]**

Write your solution to Question #4(b) below

Additional space for Question 4

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Question 5. [20 marks]

(a) State the Residue Theorem. [**10**]

Write your solution to Question #5(a) below

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(b) Using the Residue Theorem, or otherwise, compute

$$\int_C \frac{dz}{(z-5)(z+1)^4},$$

where C is the positively oriented circle of radius $\frac{5}{2}$ centred at $z = 1$. **[10]**

Write your solution to Question #5(b) below

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Additional space for Question 5

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This page is for additional work and will NOT be marked.

End of Paper.

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