MTH4107 / MTH4207 - INTRODUCTION TO PROBABILITY - 2021/22

YOU CAN PREVIEW THIS QUIZ, BUT IF THIS WERE A REAL ATTEMPT, YOU WOULD BE BLOCKED BECAUSE:

This quiz is not currently available

QUESTION 1

Not yet answered Marked out of 6.00

Boris, Priti, and Sajid are due to attend a meeting. A curious observer records their actual attendance. Abbreviating the names by initials, which of the following should be written down as the sample space for the experiment?

Select one:

- a. $\{B, P, S\}$
- b. $\{B, P, S, BP, PB, BS, SB, PS, SP, BPS, BSP, PBS, PSB, SBP, SPB\}$
- c. $\{X, B, P, S\}$
- d. None of the other choices
- e. $\{\emptyset, \{B\}, \{P\}, \{S\}, \{B, P\}, \{B, S\}, \{P, S\}, \{B, P, S\}\}$

QUESTION 2

Not yet answered Marked out of 8.00

Suppose A and B are subsets of S. Match up the following events with their complements.

$A^c \cup B^c$	Drag answer here	$A^c \cap B$
$A \triangle B$	Drag answer here	$(A^c \cap B^c) \cup (A \cap B)$
$B^c \cup A$	Drag answer here	$(A \cup B) \cap (A \cap B)$
$A \setminus B$	Drag answer here	$A^c \cup B$

QUESTION 3

Not yet answered Marked out of 6.00

On Blackchurch High Street there are two competing restaurants: Bayes' Burgers and Kolm's Kebabs. You notice that on any particular day, Bayes' Burgers is closed with probability 0.2, while Kolm's Kebabs is open with probability 0.67. Given Kolm's Kebab's is open, Bayes' Burgers is open with probability 0.88. Using the inclusion-exclusion principle, or otherwise, determine the probability that at least one of the restaurants is open. [Enter the answer correct to two decimal places.]

Answer:

The sock drawer of an eccentric lecturer contains five black socks, three red socks, and two socks with famous equations on. If she picks two socks at random to wear, what is the probability they don't match?

Select one:

- a. $8/10$
- b. $31/50$
- c. $14/45$
- d. None of the other choices
- e. $18/25$

QUESTION 5

Not yet answered Marked out of 8.00

Suppose A and B are events with $\mathbb{P}(A) = 0.25$, $\mathbb{P}(B) = 0.64$, and $\mathbb{P}(A \cup B) = 0.73$. Then A and B ______ independent and ______ disjoint.

QUESTION 6

Not yet answered Marked out of 6.00

If A and B are events which are conditionally independent given a third event C, which of the following is *not* necessarily true?

Select one:

- a. $\mathbb{P}(A \cap B|C^c) = 1 - \mathbb{P}(A|C)\mathbb{P}(B|C)$
- b. None of the other choices
- c. $\mathbb{P}(A \cap B|C) = \mathbb{P}(A|C) - \mathbb{P}(A|C)\mathbb{P}(B^c|C)$
- d. $\mathbb{P}(B|A \cap C) = \mathbb{P}(B|C)$
- e. $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A|C)\mathbb{P}(B \cap C)$

QUESTION 7

Not yet answered Marked out of 6.00

The number of goals in a football match is a Poisson random variable with parameter $\lambda = 1.54$. Given the number of goals is less than three, find the probability that there are no goals. [Enter the answer correct to two decimal places.]

Answer:

QUESTION 8

Not yet answered Marked out of 6.00

Suppose that $Y \sim \mathrm{Bin}(n, p)$ with $0 < p < 1$, and that $Z = aY$ where a is a non-zero constant. If $\mathbb{E}(Z) = \mathrm{Var}(Z)$, what is a?

Select one:

- a. $1/(1-p)$
- b. Cannot be determined without more information
- c. $1/[p(1-p)^2]$
- d. None of the other choices
- e. $p/(1-p)$

The QMplus-gremlins are mischievous creatures that cause computer problems. After careful research you observe that the number of gremlins active each day is a random variable with $\mathrm{Bin}(3, 0.5)$ distribution. If no gremlins are active, the probability of QMplus failure is 0.15; if one gremlin is active the probability of QMplus failure is 0.3; if two or more gremlins are active the probability of QMplus failure is 0.5. You conclude that the total probability of QMplus failure on a given day is ______ (to two decimal places).

Assuming there is a failure, the probability no gremlins are active is ______ (again to two decimal places).

QUESTION 10

Not yet answered Marked out of 8.00

Suppose that U and V are independent random variables with $U \sim \mathrm{Poisson}(2)$ and $V \sim \mathrm{Geom}(1/2)$. Match up the following quantities.

$\mathrm{Var}(2U)$	Drag answer here	0
$\mathbb{E}(U + V - 4)$	Drag answer here	6
$\mathbb{E}(V^2)$	Drag answer here	8
$\mathrm{Var}(U - V)$	Drag answer here	4

QUESTION 11

Not yet answered Marked out of 6.00

Suppose that Y and Z are random variables with $Y \sim \mathrm{Bernoulli}(p)$ and $Z = -Y + 1$. Which of the following gives the joint probability mass function $\mathbb{P}(Y = y, Z = z)$ for $y = 0, 1$ and $z = 0, 1$?

Select one:

- a. $yzp + (1 - y)(1 - z)(1 - p)$
- b. $(1 - y)zp + y(1 - z)(1 - p)$
- c. $y(1 - z)p + (1 - y)z(1 - p)$
- d. $(1 - y)(1 - z)p + yz(1 - p)$
- e. None of the other choices

QUESTION 12

Not yet answered Marked out of 6.00

Suppose that V and W are random variables with $\mathrm{Cov}(V, W) = 0$. Which of the following must also be true?

Select one:

- a. $\mathrm{Corr}(V + 2, 3W + 4) = 0$
- b. $-\mathbb{E}(VW) = \mathbb{E}(-V)\mathbb{E}(-W)$
- c. None of the other choices
- d. V and W are independent
- e. $\mathrm{Var}(V) = \mathrm{Var}(W)$

(a) Let N be a random variable taking values in the set $\{1, 2, 3, \ldots\}$.

(i) Write down, as a sum, a formula for the 2021st moment of N. [3 marks]

(ii) If $N \sim \mathrm{Geom}(p)$, explain in words why $\mathbb{P}(N > n) = (1-p)^n$ for any non-negative integer n. [3 marks]

(b) In 2022 you make a New Year's resolution to do some exercise every day, starting on January 1st. The number of days until you break this resolution is a geometric random variable with parameter $1/(k+1)$ where k is the last non-zero digit in *your* student ID number.

(i) Determine the expected number of days until you break the resolution. [2 marks]

(ii) Given you haven't broken your resolution by January 12th, find the probability you haven't broken it by January 14th. [5 marks]

(c) You optimistically make a second New Year's resolution to eat fruit every day, again starting on January 1st. The number of days until you break this resolution is also a geometric random variable with the same distribution as in (b).

(i) Assuming that the two random variables are independent, calculate the probability that you break both resolutions on the same day. You may find it helpful to use the formula for the sum of a geometric series. [5 marks]

(ii) Discuss briefly whether the assumption of independence is reasonable in this situation. [2 marks]

Your work must be *handwritten* and uploaded as a **single PDF file** with your student ID number on each page. You are reminded that you should show your working and explain the steps in your solutions, except where you are simply asked to "State" or "Write" a result. Your final answers should be simplified as far as possible.

Maximum file size: Unlimited, maximum number of files: 1