Queen Mary
University of London

BSc Examination by course unit 2015

MTH4105 Introduction to Mathematical Computing

Duration: 2 hours

Date and time: Friday 22 May 2015, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Answer each question in the appropriate subsection headed *Answers* in the Test answer template provided in QMplus. You must use Maple to perform all calculations. Do not delete any relevant input or output; you will score marks only for what is visible in the document you submit. There may be more than one correct solution to each question; any working solution will be accepted provided it satisfies the requirements of the question.

This exam is open book. You may access any information you want, but must work entirely by yourself. You may not communicate, nor attempt to communicate, with anyone else, nor solicit assistance in any way. Please be aware that details of all internet activity on your computer may be logged. You may do rough work on your own paper, which will not be collected by the invigilators.

Exam papers must not be removed from the examination room.

Examiners: F. J. Wright, W. Just

TURN OVER

Question 1 [12 marks]

a) Assign the polynomial $x^4 - 3x^2 + x + 1$ to the variable *y* and plot the graph of *y* against *x* for $-2 \leq x \leq 2$. **[4 marks]**

b) Assign the derivative of *y* with respect to *x* to the variable *y1* and use *y1* to find the *x* values of all the (real) stationary points of *y* as explicit real numerical approximations accurate to Maple's default precision. **[4 marks]**

c) Assign the *x* value of the local maximum of *y* near the origin to the variable *x0* and evaluate the second derivative of *y* with respect to *x* at *x* = *x0*. **[4 marks]**

Question 2 [12 marks]

Each of the following expressions contains 5 elements or terms. Construct each expression by evaluating input that uses an appropriate function such as *seq*, *add* or *mul* and could construct an expression of the form shown containing an arbitrary number of elements or terms.

a) the set $\{10, 20, 30, 40, 50\}$ **[3 marks]**

b) the product $f(1)\,f(2)\,f(3)\,f(4)\,f(5)$ **[3 marks]**

c) the list of lists $[[1, 2], [2, 3], [3, 4], [4, 5], [5, 6]]$ **[3 marks]**

d) the sequence $f(x),\ \frac{d}{dx} f(x),\ \frac{d^2}{dx^2} f(x),\ \frac{d^3}{dx^3} f(x),\ \frac{d^4}{dx^4} f(x),\ \frac{d^5}{dx^5} f(x)$ **[3 marks]**

Question 3 [12 marks]

a) Assign the set of integers from 1 to 100 inclusive to the variable *A*. Use *A* to assign the set whose elements are each twice an element of *A*, i.e. the set $\{2r : r \in A\}$, to the variable *B*. Use the variables *A*, *B* to answer the rest of this question. **[5 marks]**

b) Assign the set *A* minus the set *B* to the variable *AB* and assign the set *B* minus the set *A* to the variable *BA*. **[3 marks]**

c) Show that the sets *AB* and *BA* are different but have the same cardinality by entering and evaluating a single expression that evaluates to *true*. **[4 marks]**

Question 4 [14 marks]

a) A sequence of rational numbers can be defined recursively by $x_0 = 1$, $x_n = 1 + \frac{1}{x_{n-1}}$, $n > 0$. Use a **do** loop to construct this sequence for $0 \leq n \leq 10$. **The output should consist of the sequence and nothing else. [7 marks]**

b) Modify your answer to part (a) to construct a list of lists of the form $[[0, x_0], [1, x_1], [2, x_2], \ldots, [10, x_{10}]]$ and plot this list as points with appropriately labelled axes and appropriate axis ranges. **The output should consist of the plot and nothing else. [7 marks]**

Question 5 [12 marks]

Let $A = \{-2, -1, 0, 1, 2\}$.

a) Define the continuous formula-based function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $x \mapsto x^2 + x + 1$. **[3 marks]**

b) Define the discrete table-based function $g : A \rightarrow \mathbb{Q}$ such that $-2 \mapsto 3, -1 \mapsto 1, 0 \mapsto 1, 1 \mapsto 3, 2 \mapsto 7$. **[3 marks]**

c) Show that *f* restricted to be a function $A \rightarrow \mathbb{Q}$ is equal to *g* by entering and evaluating a single expression that evaluates to *true*. **[3 marks]**

d) Show that *g* is not injective (one-to-one) by entering and evaluating a single expression that evaluates to *false*. **[3 marks]**

Question 6 [12 marks]

a) Let $A = \{1, 2, 3, 4, 5\}$ and let &*R* be a relation on *A* such that 1 &*R* 1, 2 &*R* 2, 3 &*R* 3, 4 &*R* 4, 5 &*R* 5 (i.e. the relation &*R* is true when the two operands are the same) and no other elements of *A* are related by &*R* (i.e. the relation &*R* is false for all other ordered pairs of operands). Implement the relation &*R* in Maple as a neutral operator. **[9 marks]**

b) Assuming that &*R* is an equivalence relation, construct the set of its equivalence classes. **[3 marks]**

Question 7 [14 marks]

a) Write a **procedure** called *intset* that takes two arguments, *maxint* and *m*, both of which you can assume to be positive integers, and returns the set of integers between 1 and *maxint* inclusive minus any integer multiples of *m*. For example, it should work like this:

$intset(20, 5) = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19\}$

Declare local variables as appropriate and test your procedure. **[7 marks]**

b) Now modify *intset* so that it takes two arguments, *maxint*, which you can assume to be a positive integer and *minusset*, which you can assume to be a set of positive integers, and returns the set of integers between 1 and *maxint* inclusive minus any integer multiples of any elements of *minusset*. For example, it should work like this:

$intset(20, \{3, 5, 7\}) = \{1, 2, 4, 8, 11, 13, 16, 17, 19\}$

Declare local variables as appropriate and test your procedure. **[7 marks]**

Question 8 [12 marks]

Let $z_1 = 1 + 2i$ and $z_2 = 2 + 3i$, where i denotes the imaginary unit. (You are not required to use any specific variables.)

a) Show that z_1 and z_2 satisfy the triangle inequality $|z_1| + |z_2| \geq |z_1 + z_2|$ by entering and evaluating an expression that evaluates to *true*. **[2 marks]**

b) Show that z_1 and z_2 satisfy the two identities $|z_1 z_2| = |z_1| \, |z_2|$ and $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ by entering and evaluating two expressions that evaluate to *true*. **[4 marks]**

c) Show that z_1 and z_2 probably satisfy the two identities

$arg(z_1 z_2) = arg(z_1) + arg(z_2)$ and $arg\left(\frac{z_1}{z_2}\right) = arg(z_1) - arg(z_2)$, where *arg* denotes the argument of a complex number, by showing that the difference between the left and right hand sides of each of the identities is numerically very small in magnitude. **[6 marks]**

End of Paper