Queen Mary University of London

Main Examination period 2019

MTH4101 / MTH4201: Calculus II

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiners: R. J. Harris, R. Buzano

For Question 1 of this exam, only your final answers will be marked; partially correct answers will be awarded partial marks. For all other questions, you should show your working and carefully justify every step, except where you are simply asked to "State" or "Write down" a result.

Question 1. [40 marks]

(a) Find the first three terms and the sum of the series

$$\sum_{n=1}^{\infty} \left(\frac{3}{n} - \frac{3}{n+1} \right).$$

[5]

(b) List three tests which can be used to show that a series diverges. Which of those tests can also be used to show convergence? [5]

(c) Find the Maclaurin series for the function

$$f(x) = e^{-x},$$

explicitly including terms up to order x^3. [5]

(d) Find the natural domain for the function

$$g(x, y) = \sum_{n=0}^{\infty} \left(\frac{x}{2y} \right)^n.$$

[5]

(e) For the function

$$h(x, y) = xe^y + y + 1,$$

compute the partial derivatives h_x, h_y, h_{xx}, and h_{yy}. [5]

(f) Write a chain rule formula for $\partial w / \partial r$ where w is a function of the intermediate variable u which is itself a function of r, s, and t. [5]

(g) Evaluate the double integral

$$\int_0^{\pi} \int_{-2}^{0} \left(\frac{x}{2} + \sqrt{y} \right) dx\, dy.$$

[5]

(h) Find the Jacobian $\partial(x, y, z)/\partial(\rho, \phi, \theta)$ for the transformation $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. [5]

Question 2. [20 marks]

(a) Write down the correct statement to complete the following definition:

> "The sequence $\{a_n\}$ **converges** to the number L if for every positive number ϵ there corresponds an integer N such that for all n, ...".

[5]

(b) Consider the sequence defined by first term $b_1 = 2$ and recursion formula

$$b_{n+1} = \frac{b_n - 1}{b_n + 3}.$$

(i) Find the values of b_2, b_3, and b_4. **[5]**

(ii) Determine the limit of the sequence. [You may assume it converges.] **[6]**

(iii) Does the series $\sum b_n$ converge or diverge? Explain your reasoning. **[4]**

Question 3. [20 marks]

(a) State the definition of a **critical point** (for a function of two variables). **[3]**

(b) Consider the function

$$g(x, y) = 2e^x \sin y.$$

(i) Prove that there are no critical points of g. **[6]**

(ii) Calculate the gradient vector ∇g at the point $(0, \pi/4)$. Use your result to calculate the derivative of g at $(0, \pi/4)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$. **[8]**

(iii) Is there a direction $\mathbf{w}$ in which the derivative of g at $(0, \pi/4)$ is equal to 3? Give reasons for your answer. **[3]**

Question 4. [20 marks]
The area of a region R in the xy-plane is given by

$$A = \int_0^2 \int_{x^2/4}^{(x+2)/4} dy\, dx.$$

(a) Evaluate the above integral to find the area, A. **[5]**

(b) Sketch the region R. **[4]**

(c) Write down, but do not evaluate, an expression for the area A with the order of integration reversed. **[5]**

(d) Find the average value of $f(x) = x^2$ over the region R. **[6]**

End of Paper.