Main Examination period 2018

MTH4101 / MTH4201: Calculus II

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: R. J. Harris, J. Ward

For Question 1 of this exam, only your final answers will be marked; partially correct answers will be awarded partial marks. For all other questions, you should show your working and carefully justify every step, except where you are simply asked to "State" or "Write down" a result.

Question 1. [40 marks]

(a) Find the first two terms and the sum of the series

$$\sum_{n=0}^{\infty} \frac{5}{2^n}.$$

[5]

(b) State whether the series

$$\sum_{n=1}^{\infty} \frac{1}{n+4}$$

diverges or converges, indicating which test justifies your answer. **[5]**

(c) Find the Taylor polynomials of orders 0, 1, and 2 for the function

$$f(x) = \cos(-x),$$

about the point $x = 0$. **[5]**

(d) Evaluate

$$\lim_{\substack{(x,y)\to(2,2) \\ x\neq y}} \frac{x^2 - y^2}{x - y}$$

or state that the limit does not exist. **[5]**

(e) For the function

$$g(x, y) = e^{xy} \ln y,$$

compute g_x and g_y. **[5]**

(f) Calculate the gradient vector of

$$h(x, y, z) = 3e^{-x} + \sin(2y) + 4\sqrt{z+1}$$

at the point $(0, 0, 0)$. **[5]**

(g) Transform the integral

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)^3 \, dx \, dy$$

into an equivalent polar integral. [Evaluation of the integral is not required here.] **[5]**

(h) Evaluate the triple integral

$$\int_{\pi/4}^{\pi/2} \int_0^{\pi} \int_{-e}^{e} y \sin z \, dx \, dy \, dz.$$

[5]

Question 2. [20 marks]

(a) Consider the following (erroneous) definition.

> "The sequence $\{a_n\}$ **diverges to infinity** if for some number M there is an integer N such that for all n larger than N, $a_n > M$."

(i) Identify and correct the error in the above definition. **[2]**

(ii) Write down a correct definition for a sequence which **diverges to negative infinity**. **[2]**

(b) A sequence is defined recursively by

$$b_1 = -1, \quad b_{n+1} = \frac{4n-5}{2n-3} b_n.$$

(i) Find the values of b_2, b_3, b_4, and b_5. **[5]**

(ii) State whether this sequence is monotonic and whether it is bounded. **[2]**

(iii) State whether the sequence diverges to infinity, diverges to negative infinity, or neither. [A proof is not necessary here.] **[2]**

(iv) Use the Ratio Test to show whether the series $\sum_{n=1}^{\infty} b_n$ converges or diverges. **[7]**

Question 3. [20 marks]

(a) State the **First Derivative Test for Local Extreme Values** (for a function of two variables). **[3]**

(b) Find all critical points of the function

$$f(x, y) = x^2 - 3xy + 3y^2 - 3x + 3y + 1.$$

Identify whether each point you find is a local maximum, local minimum, or saddle point, and find the value of the function there. **[10]**

(c) Find the two numbers u and v, with $u \leq v$, for which

$$\int_u^v (6 - x - x^2)\, dx$$

has its largest value. **[7]**

Question 4. [20 marks]

(a) Solve the system $u = 2x$, $v = y - 2x$ to find expressions for x and y in terms of u and v. Use these expressions to find the Jacobian $\partial(x, y)/\partial(u, v)$. **[6]**

(b) Consider the integral

$$\int_0^1 \int_{2x}^{2x+1} (2y - 4x)\, dy\, dx.$$

(i) Sketch the region of integration. **[3]**

(ii) Use the transformation in (a) to evaluate the integral. **[8]**

(iii) Demonstrate that you get the same answer by integrating directly with respect to y and then x. **[3]**

End of Paper.