Queen Mary
University of London

B. Sc. Examination by course unit 2014

MTH4101: Calculus II

Duration: 2 hours

Date and time: 8 May 2014, 10:00h–12:00h

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You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): R. Klages, Y. Fyodorov

Question 1

(a) Consider the function

$$f(x,y) = \frac{2x - xy - 3y + 6}{y - 2}\,, \; y \neq 2\,.$$

Find the limit of f as $(x, y) \to (1, 2)$. [7]

(b) Find the directional derivative of the function

$$f(x,y,z) = xy + xz + yz$$

at the point $(-1, 1, -2)$ in the direction of the vector $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ [7]

(c) Find the linearisation of

$$f(x,y) = 7x + 3y - 11$$

at the point $(2, 3)$. Justify your answer. [7]

(d) Obtain the limit as $n \to \infty$ for the sequence

$$a_n = \left(1 + \frac{5}{n}\right)^n\,.$$

[7]

(e) Find the sum of the series

$$\sum_{k=0}^{\infty} (-2)^k \frac{3}{4^k}\,.$$

[7]

(f) Sketch the region of integration, and then reverse the order of integration, to evaluate the integral

$$\int_0^1 \int_y^{\sqrt{y}} (3x - 1)\; dx\; dy\,.$$

[7]

(g) Find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$ for the transformation $x = v + 2w, y = u + vw, z = uv^2$. [7]

(h) Solve the differential equation

$$\frac{dy}{dx} = e^{y-x}$$

by giving the solution in implicit form. [7]

Question 2

(a) Define a *level curve* for a function f of two variables x and y. [2]

(b) Write down the equation of the *tangent line* through the point (x_0, y_0) to a level curve of such a function f. [3]

(c) Sketch the level curve

$$\frac{x^2}{4} + y^2 = 2 \quad .$$

Calculate the gradient vector ∇f to this curve at the point $(2, -1)$ and sketch it. Then calculate the equation for the tangent line to the level curve at the same point and sketch it. [6]

Question 3 Use the method of Lagrange multipliers to find the extreme points of the function

$$f(x, y) = xy$$

subject to the condition

$$x^2 + y^2 = 9 \, .$$

Determine the value of the function f at each of the extreme points. [11]

Question 4 Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n} \quad .$$

Find the values of x for which the series converges absolutely and conditionally. [11]

Question 5

(a) Let f be a function with derivatives of all orders throughout some interval containing a as an interior point. Define the *Taylor series* generated by f at $x = a$. [4]

(b) Find the Taylor series generated by $f(x) = e^{2x}$ at $x = 0$. [7]

End of Paper