MTH4100: Calculus I

Duration: 2 hours

Date and time: 9 January 2018

Student ID:

Name:

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You should attempt ALL questions. Marks awarded are shown next to the questions.
Write your calculations and answers in the boxes provided and perform additional calculations on the blank pages at the end of the booklet.

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Examiner(s): O.F. Bandtlow and W. Just

Question 1. Find the natural domain and the corresponding range of the function   [8]

$$f(x) = \frac{x+1}{|x+1|}.$$

Question 2. Find the limit   **[8]**

$$\lim_{x \to 0} \frac{\sin(2x)}{\sin(3x)} \ .$$

Question 3. At which points is the following function continuous?   **[8]**

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & |x| \neq 2; \\ 4, & |x| = 2. \end{cases}$$

Question 4. Compute the derivative of the following function [8]

$$f(x) = \frac{1}{1 + \cos(x^3)}.$$

Question 5. Find the equation of the tangent to the graph of   [8]

$$f(x) = \ln(x^2 + x^4)$$

at $x = -1$.

Question 6. Find the absolute maximum and minimum of [8]

$$f(x) = \frac{x}{x^2 + 4}$$

on $[-1, 4]$.

Question 7. Determine all critical points for the following function   [8]

$$f(x) = \sqrt{2x - x^2}\,.$$
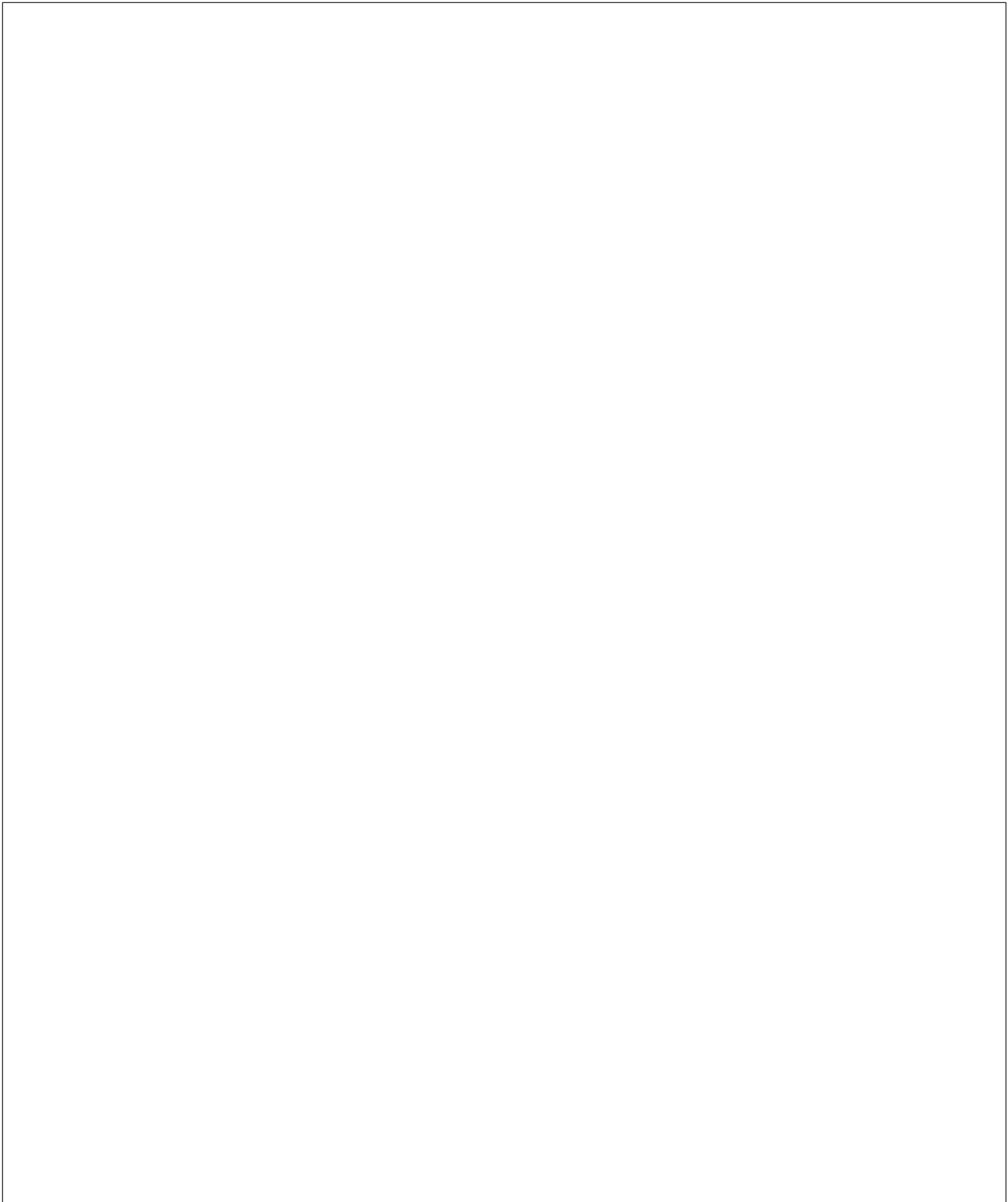
Question 8. Find all inflection points of [8]

$$f(x) = x^4 - 8x^2 + 6x - 3$$

and determine the open intervals on which the graph of f is concave up or concave down.

Question 9. Graph the function   [8]

$$f(x) = \ln(|x| + 1).$$

Question 10. Suppose that f is a continuous function on $[0, \infty)$ with [8]

$$\int_0^x f(t)\,dt = x\sin(\pi x)$$

for all $x \geq 0$. Find $f(1)$.

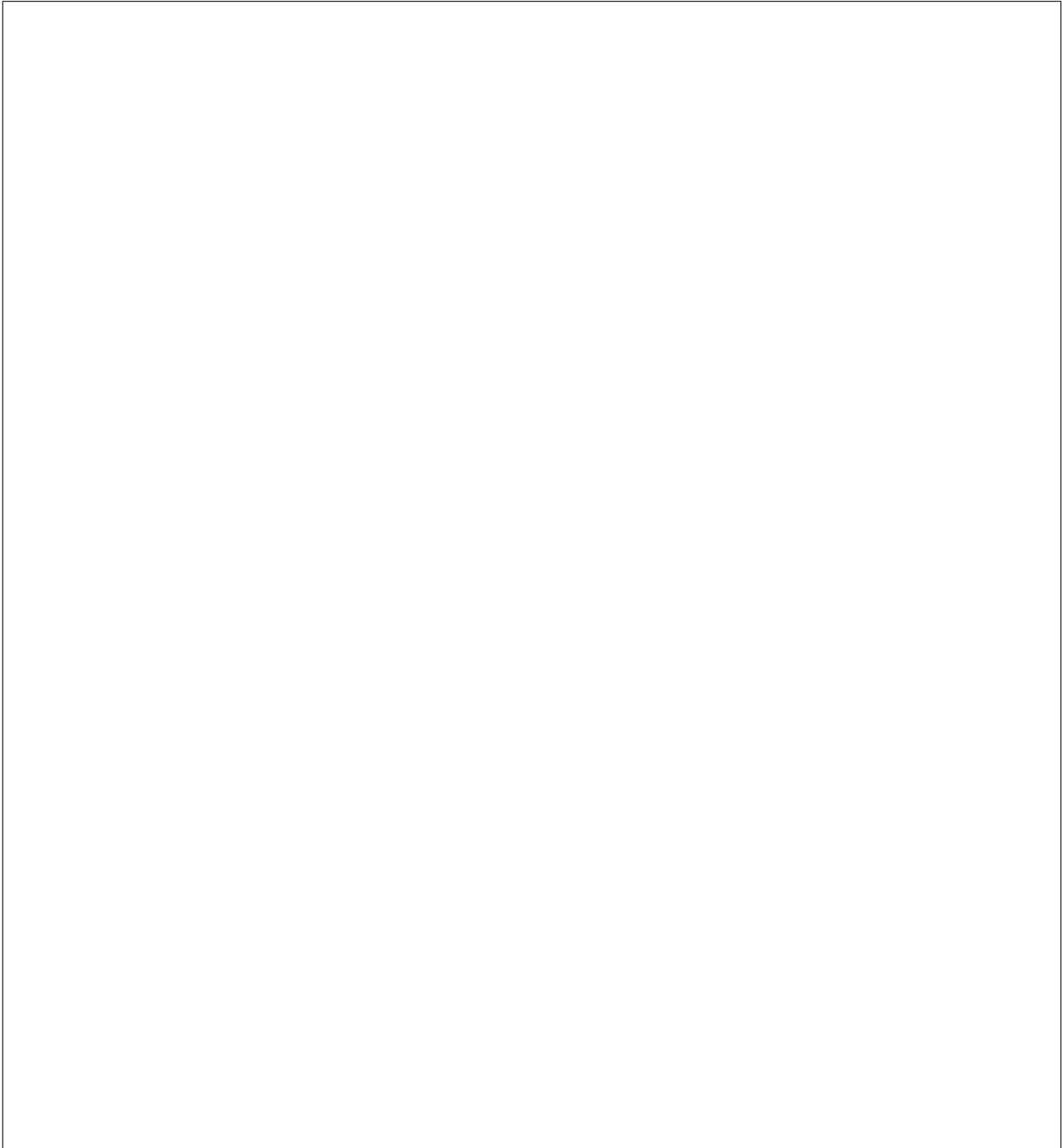**Question 11.** Evaluate the following indefinite integral [10]

$$\int \exp(\sin(x)) \sin(2x)\, dx\,.$$

Question 12. Evaluate the improper integral [10]

$$\int_0^{\pi/2} \frac{\exp(-\tan(x))}{\cos^2(x)} \, dx \,.$$

End of Paper.

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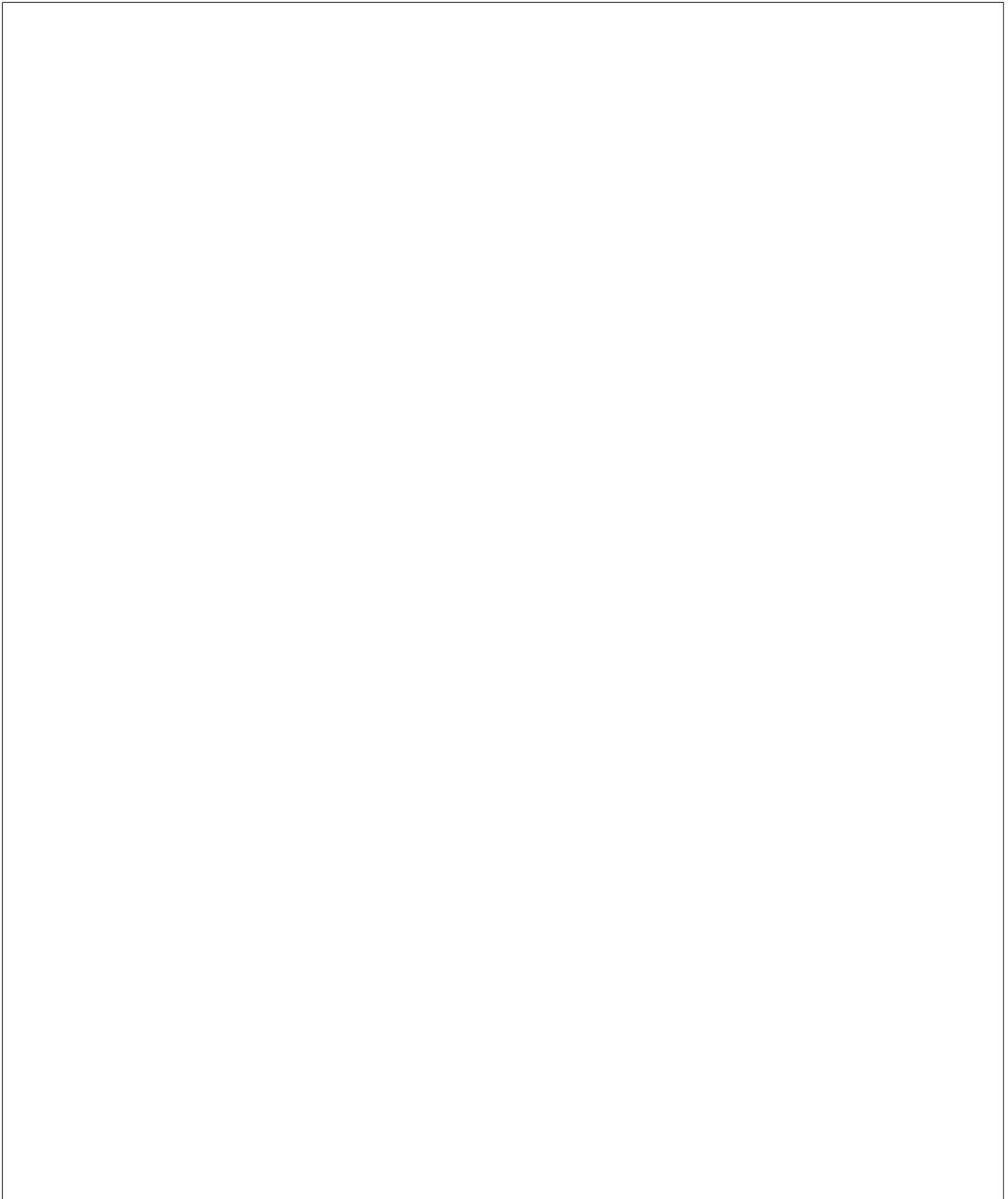
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Turn Over

Question 1. Graph the function   [8]

$$f(x) = \exp(-|x+1|).$$

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Question 2. Find all values of t for which [8]

$$\cos(2t) + 3 = 4\cos(t).$$

Question 3. Use limits to determine the equations of all horizontal asymptotes of the function [8]

$$f(x) = \frac{x - 10}{\sqrt{4x^2 + 5}}.$$

Question 4. Find the value of c such that the following function is continuous on $\mathbb{R}$ [8]

$$f(x) = \begin{cases} \frac{\arctan(2x)}{x}, & x \neq 0; \\ c, & x = 0. \end{cases}$$

Question 5. Using the definition of the derivative of a function as a limit, find the derivative of   **[8]**

$$f(x) = 4 - x^2.$$

Question 6. Compute the derivative of the following function [8]

$$f(x) = x \exp(\tan(x)).$$

Question 7. Determine the linearisation of the function [8]
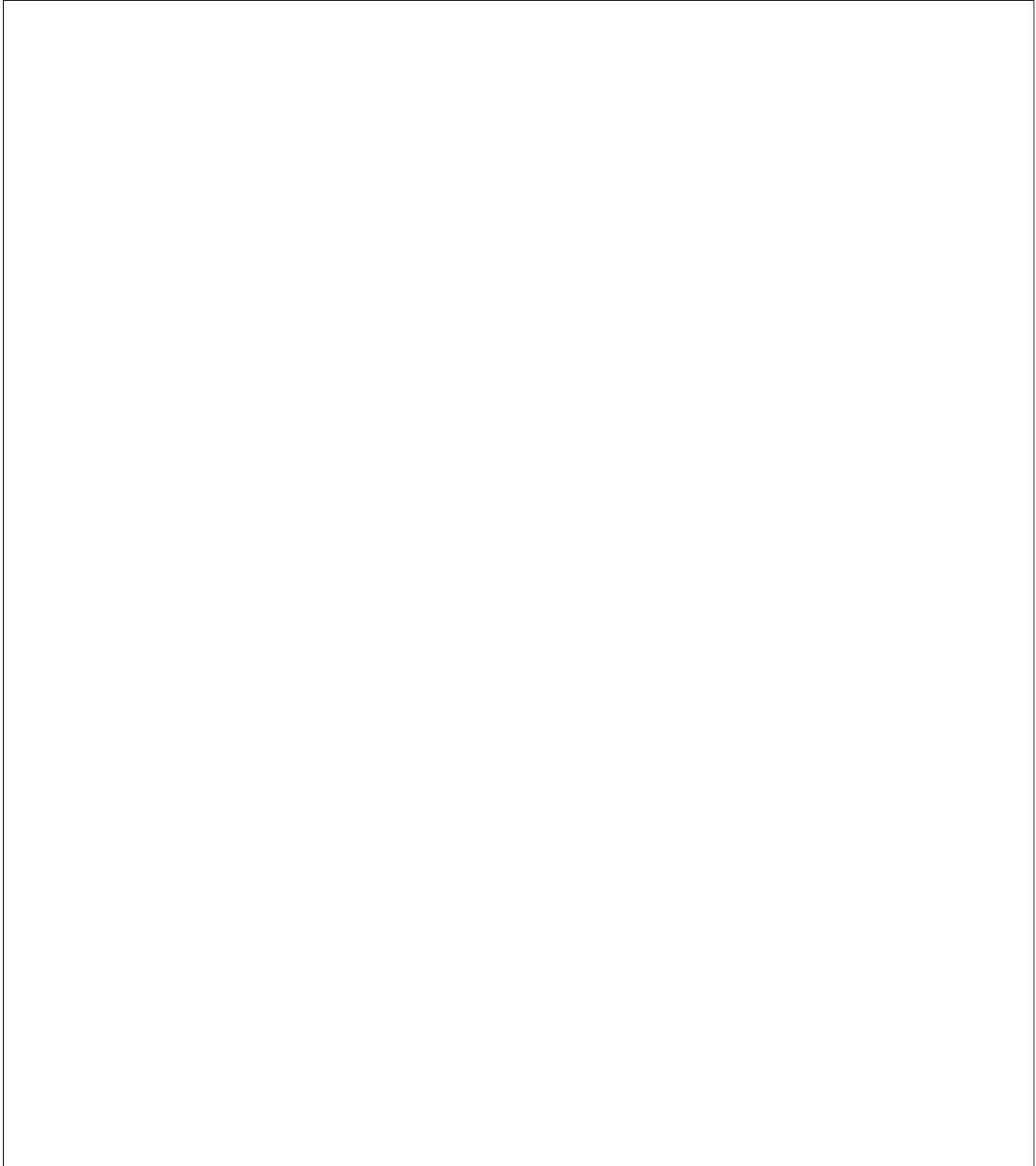
$$f(x) = \ln(1 + x^2)$$

at $x = -2$.

Question 8. Show that the function [8]

$$f(x) = x^3 - x - \cos(\pi x)$$

has at least two critical points in $[-1, 1]$.

Question 9. Let f be given by   [10]

$$f(x) = 3 + 15x^3 - x^5.$$

Determine the open intervals on which f is increasing or decreasing.

Question 10. Calculate the definite integral [8]

$$\int_0^{2\pi} |\sin(x)|\, dx.$$
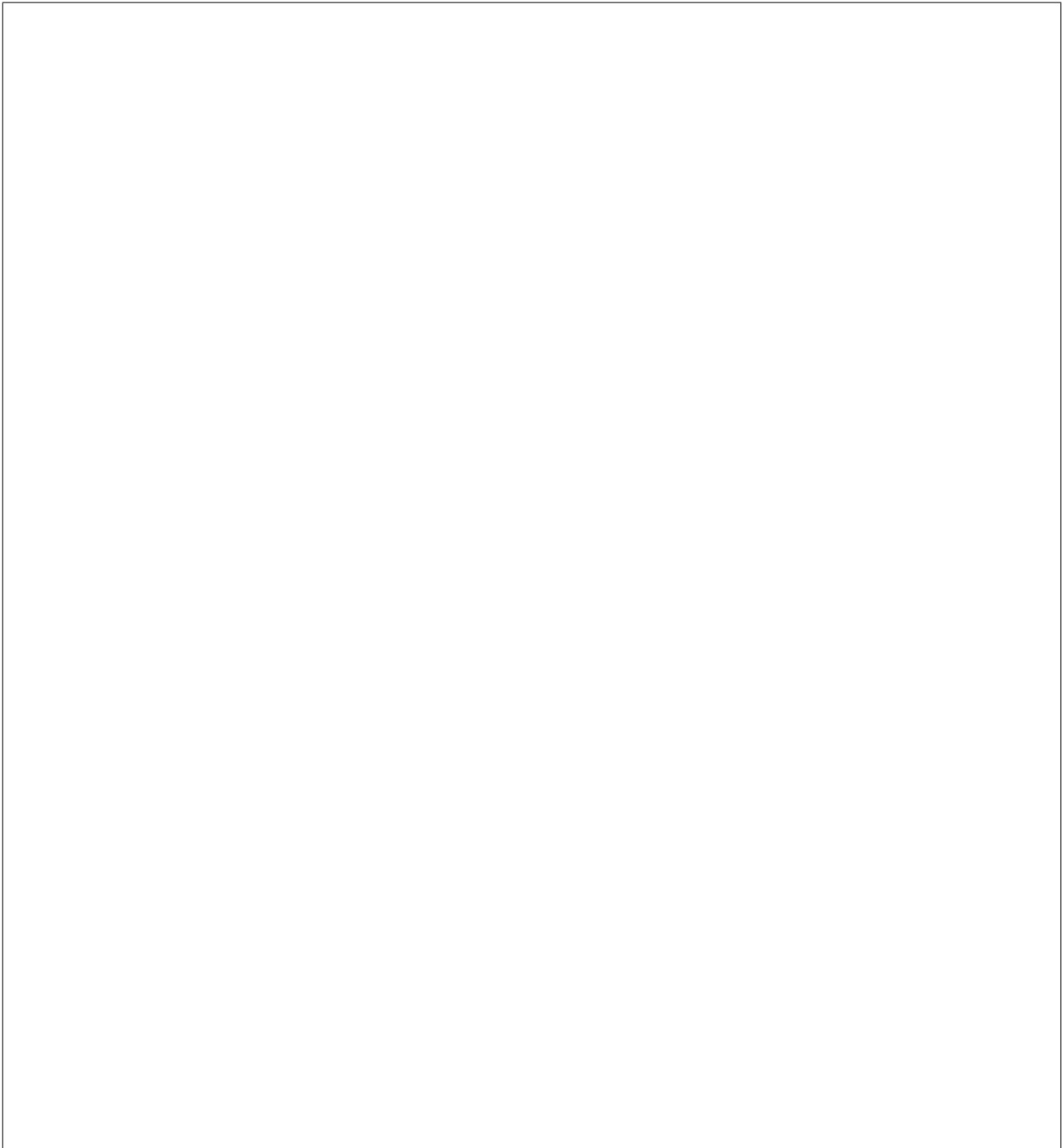
Question 11. Find all antiderivatives of the function   [8]

$$f(x) = x\cos(2x).$$

Question 12. Evaluate the improper integral [10]

$$\int_{-\infty}^{\infty} \exp(-|x+1|)\, dx\,.$$

End of Paper.