MTH4100: Calculus 1

Duration: 2 hours

Date and time: 11 May 2016, 10:00h–12:00h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiner(s): Prof. B. Jackson

Question 1. (a) Let

$$f(x) = \frac{x^2 - x - 2}{|x-2|}\,.$$

Determine whether the following limits exist, giving brief justifications for your answers:

$$\lim_{x\to 2^+} f(x); \qquad \lim_{x\to 2^-} f(x); \qquad \lim_{x\to 2} f(x).$$

[**5 marks**]

(b) Let

$$f(x) = \begin{cases} x^{-1} & \text{if } x \le 1 \\ x^2 & \text{if } x > 1. \end{cases}$$

Determine whether f is differentiable at $x = 1$, giving a brief justification for your answer. [**5 marks**]

(c) Find the derivative of $h(x) = \sin(\ln 2x)$. [**5 marks**]

(d) Find all asymptotes of the graph of

$$f(x) = \frac{2x^2 + x + 1}{x+1}$$

and describe the behaviour of f at each asymptote. [**7 marks**]

(e) Given that

$$y^2 + xy + 3 = 0\,,$$

and $y = y(x)$, find the values of $\mathrm{d}y/\mathrm{d}x$ and $\mathrm{d}^2y/\mathrm{d}x^2$ when $y = 1$. [**8 marks**]

(f) Find the area enclosed by the curves $x = 0$, $y = 2$ and $y = e^x$. [**7 marks**]

(g) Evaluate

$$\int x \sec^2 x \,\mathrm{d}x\,.$$

[**8 marks**]

Question 2. Consider the function

$$f(x) = x^2 + \frac{2}{x}\,.$$

(a) Identify the domain of f and determine whether or not f is an even function or an odd function. **[2 marks]**

(b) Find $f'(x)$ and $f''(x)$. **[4 marks]**

(c) Find the critical points of f, determine where f is increasing or decreasing, and determine the behaviour of f at each of its critical points. **[7 marks]**

(d) Determine the values of x for which the graph of $y = f(x)$ is concave up and the values for which it is concave down, and find any points of inflexion. **[5 marks]**

(e) Determine the behaviour of $f(x)$ as $x \to \pm\infty$ and identify any asymptotes. **[2 marks]**

(f) Plot key points, such as intercepts, critical points, and points of inflexion, and sketch the graph. **[5 marks]**

Question 3. Let f be a function defined on the interval $[0,1]$.

(a) Explain what is meant by a *Riemann sum* for f over $[0,1]$ with respect to the partition of $[0,1]$ into n equal subintervals. **[3 marks]**

(b) Explain what it means to say that f is *integrable* on $[0,1]$. **[2 marks]**

Question 4. Define the improper integral $\int_0^1 x^{-\frac{1}{2}}\,\mathrm{d}x$ as a limit of definite integrals and evaluate it. **[10 marks]**

Question 5. (a) State, without proving, both parts of the Fundamental Theorem of Calculus. **[7 marks]**

(b) Let

$$H(x) = \int_0^x e^t \cos t\,\mathrm{d}t \qquad \text{and} \qquad G(x) = \int_0^{x^3} e^t \cos t\,\mathrm{d}t\,.$$

Determine $H'(x)$ and $G'(x)$. **[8 marks]**

End of Paper.