Queen Mary
University of London

Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH6113: Mathematical Tools for Asset Management

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**, and should **include your student number**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work:

- scan your work, convert it to a **single PDF file** and upload this using the upload tool on the QMplus page for the module;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions
This module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, you must submit within the first **3 hours** of the assessment period. You may then submit a second version later in the assessment period if you wish, which will count only towards your degree. There are two separate upload tools on the QMplus page to enable you to submit a second version of your work.

Examiners: K. Glau, L. Wunderlich

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Question 1 [10 marks]. The May 6, 2010, flash crash was a United States trillion-dollar stock market crash, which lasted for approximately 36 minutes. Just prior to the flash crash, a trader, Nav Sarao, placed orders for thousands of E-mini S&P 500 stock index futures contracts which he planned on cancelling later. These orders amounting to about "200 million worth of bets that the market would fall" were "replaced or modified 19,000 times" before they were cancelled. Sarao and his company, Nav Sarao Futures Limited, allegedly made more than $40 million in profit from trading from 2009–2015.
We assume his investment strategy during 2009–2015 was the same as during the crash, i.e. he illegally placed large orders which he cancelled later. Only he knew that the orders would be cancelled. Under the assumption of the semi-strong market hypothesis, is it possible that he thus consistently yields higher profit than the market, or was it simply luck?

A. Market efficiency has been developed under the premise that market manipulation is not possible. Therefore we can not answer this question.

B. The semi-strong form of market efficiency states that investments based on public information cannot consistently beat the market. His investment was based on private information, so the higher profit is not ruled out by the semi-strong form.

C. The semi-strong form of market efficiency implies the strong form. The latter implies that no investment can ever consistently beat the market. Therefore his investment was purely lucky.

D. The semi-strong form of market efficiency states that investments based on public information can not lead to gains significantly higher than the market average at any point in time. Therefore the high profit of $40 million disproves the semi-strong form of market efficiency.

E. The investment strategy is completely in line with the three hypotheses of market efficiency. Thus it underpins the strong form and therefore also the semi-strong form of the efficient market hypothesis.

(a) Write down the letter that you select. [**3**]

(b) Explain your choice expanding the argument. Do not argue against the other possible choices, use at most four sentences and not more than 70 words. [**7**]

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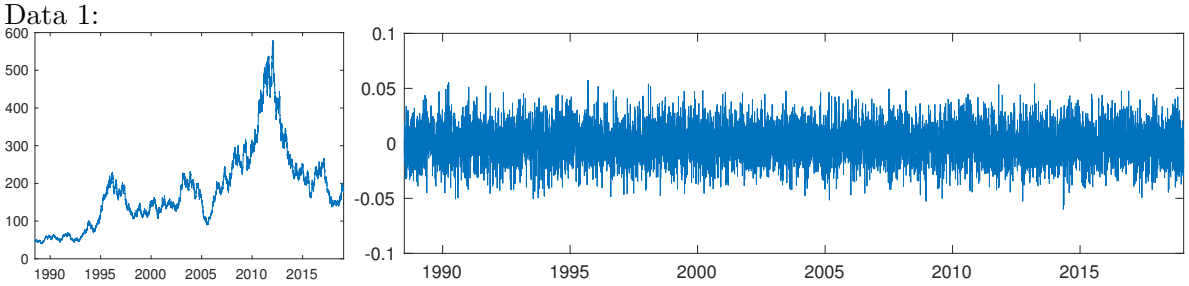
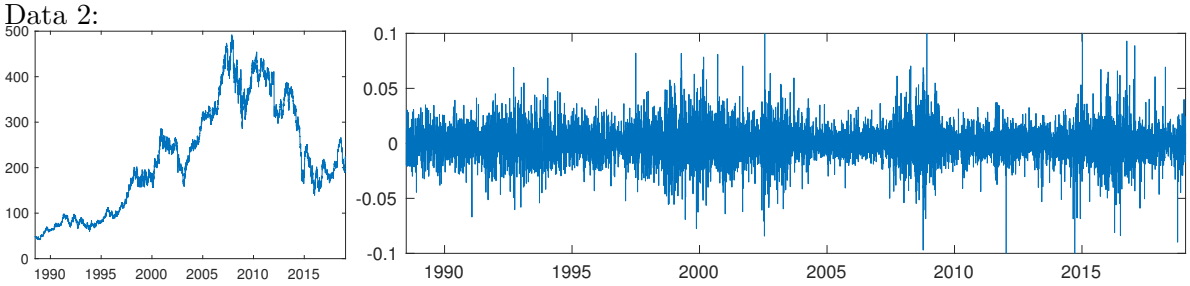
Question 2 [10 marks]. Consider the following plot of stock prices (left) and daily returns (right). One data set represents the empirical data and the other the realisation of a lognormal model.

Data 1:



Data 2:



Consider the following two statements:

I. In the lognormal model we expect to see no volatility clustering, as subsequent returns are modelled to be independent of each other. However, market data typically shows volatility clusters. We also expect to see less spikes in the model data than in the empirical one.

II. Both model and empirical data show volatility clustering, e.g., when the stock price is large in the period around 2010. However, the returns in the lognormal model are independent but uncorrelated while empirically we typically find that returns are correlated but dependent.

Which one of the statements A, B, C or D is true, when comparing both data sets?

A. Statement I. is correct and therefore we conclude that Data 1 is the empirical data and Data 2 is simulated from the lognormal model.

B. Statement I. is correct and therefore we conclude that Data 2 is the empirical data and Data 1 is simulated from the lognormal model.

C. Statement II. is correct and therefore we conclude that Data 1 is the empirical data and Data 2 is simulated from the lognormal model.

D. Statement II. is correct and thereforewe conclude that Data 2 is the empirical data and Data 1 is simulated from the lognormal model.

(a) Write down the letter that you select. [**3**]

(b) Explain your choice expanding the argument. Do not argue against the other possible choices, use maximally four sentences and not more than 70 words. [**7**]

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Question 3 [26 marks].

1. For a given portfolio, time horizon T, the regulator asks us to provide the maximum possible loss at the time T, after we exclude all worse outcomes whose combined probability is at most 5%.

   For a model of the stock prices, approved by the regulator, the parameters are already estimated.

   Which of the following should you compute?

   The daily returns of the portfolio need to be simulated until T. Then we compute

   A. the empirical variance of 5%.

   B. the empirical semi-variance of 5%.

   Using the model, the portfolio value at T is expressed as a random variable. Then we compute the related

   C. shortfall probability with threshold $b = 5\%$.

   D. value at risk with $\alpha = 95\%$.

   (a) Write down the letter that you select. [**3**]

   (b) Explain your choice expanding the argument. Do not argue against the other possible choices, use maximally four sentences and not more than 70 words. [**7**]

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2. Which comment on the quality of the lognormal model is correct?

   A. The lognormal model is one of the models that captures the heavy tails of the stocks, i.e. the high probability of large gains and losses.

   B. The model is useful in practice but carries some risks. For example, it underestimates the probability of large losses.

   C. The model is useful in practice but carries some risks. For example, it systematically underestimates the risk measured in the variance.

   D. It is hard to predict stock prices. The lognormal model is used to predict volatility clusters in practice.

   (a) Write down the letter that you select. [**2**]

   (b) Explain your choice expanding the argument. Do not argue against the other possible choices, use maximally four sentences and not more than 70 words. [**6**]

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3. Besides the lognormal model, stochastic volatility models (e.g., the autoregressive model) were also considered in the lectures. Which statement is correct?

   A. Stochastic volatility models can improve the model, as they can show volatility clusters.

   B. Stochastic volatility models can improve the lognormal model, as they reflect the fact that subsequent stock prices are highly dependent.

   C. Stochastic volatility models allow a better fit to market data. This is due to capturing spikes by jumps of the stock.

   (a) Write down the letter that you select. [**2**]

   (b) Explain your choice expanding the argument. Do not argue against the other possible choices, use maximally four sentences and not more than 70 words. [**6**]

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Question 4 [10 marks].

(a) Let x denote the number consisting of the last two digits of your student ID and let $N = 300 + x$. Assume daily log-returns X_t to be independent and identically distributed: $X_t \sim \mathcal{N}(\mu, \sigma^2)$. Under this assumption, what are the mean value and the variance of the annual log-returns $\sum_{i=1}^{N} X_i$ in terms of σ and μ? (We assume three hundred trading days per year.) Briefly justify your answer. [**6**]

(b) What does the result of (a) imply for the stock value? [**4**]

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Question 5 [24 marks]. Let x denote the number consisting of the last three digits of your student ID. You are responsible for a portfolio that consists of $£(67\,000 + x)$ invested in a risk-free asset with zero interest rate, 5 stocks of company A and 1 000 stocks of company B. The stock-prices are currently $S_0^{\mathrm{A}} = £25\,000$ for company A and $S_0^{\mathrm{B}} = £19.50$ for company B.

(a) What is the portfolio's current value? [**4**]

(b) The portfolio should always have at most 75% of its value invested in the risky assets. Is this criterion fulfilled at the current time? Justify your answer by computing the current proportion invested in the risky assets. [**6**]

(c) Assume for this sub-question the condition of part (b) applies. The portfolio will need rebalancing when:

A. the stock price of company A gets too large and that of company B gets too low;

B. the sum of the stock prices of companies A and B remains the same;

C. the sum of the stock prices of companies A and B gets too low.

D. the sum of the stock prices of companies A and B gets too large.

Write down the letter that you select. [**2**]

(d) Derive the variance of the return of the portfolio in terms of the variances of the returns of the single assets $\mathrm{Var}(R^{\mathrm{A}})$, $\mathrm{Var}(R^{\mathrm{B}})$ and their covariance $\mathrm{Cov}(R^{\mathrm{A}}, R^{\mathrm{B}})$.

Carefully justify all steps in your derivation. To do so, first **derive** an expression for the portfolio value using S_0^A, S_0^B, R^A and R^B. With its help write the portfolio return depending on R^A and R^B. Use this expression to derive and calculate the variance of the return of the portfolio.

[**12**]

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Question 6 [16 marks]. Let x denote the number consisting of the last digit of your student ID. Consider a market where all assumptions of the CAPM with a return of the risk-free asset of $\mu_0 = 1 + x\%$ are satisfied. We observe that the expected return of the market portfolio is $\mu_{\mathrm{M}} = \mathbb{E}(R^{\mathrm{M}}) = 15\%$ and its standard deviation is $\sigma_{\mathrm{M}} = \sqrt{\mathrm{Var}(R^{\mathrm{M}})} = 10\%$.

(a) Does a portfolio with expected return of 15% and a standard deviation of 5% exist? Briefly justify your answer. [**4**]

You would like to create an efficient portfolio with less risk, more specifically with a standard deviation of $\sigma_{\mathrm{P}} = \sqrt{\mathrm{Var}(R^{\mathrm{P}})} = 2\%$.

(b) What is the expected return of this efficient portfolio? Carefully justify all steps in your derivation.

Hint: It could be helpful to first derive β (the Beta of the portfolio), and then to express the expected return in terms of β and μ_{M}. [**8**]

(c) How is this portfolio constructed? [**4**]

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Question 7 [4 marks].
For each of the following functions, decide whether it is a valid utility function with domain $\mathbb{R}$ for a risk-averse investor and justify your answer.

(a) $u(x) = x$,

(b) $u(x) = e^{x^2}$,

(c) $u(x) = 2 - 0.6 \times e^{-0.75x}$.

[**4**]

End of Paper.

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