

Main Examination period 2019

MTH4116/MTH4216: Probability and Statistics I

Duration: 2 hours

Student number

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Desk number

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Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more paper, ask an invigilator for an additional booklet and attach it to this paper at the end of the exam.

You should attempt ALL questions. Marks available are shown next to the questions.
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Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.

The New Cambridge Statistical Tables are provided.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiners: H. Maruri-Aguilar, D. S. Coad

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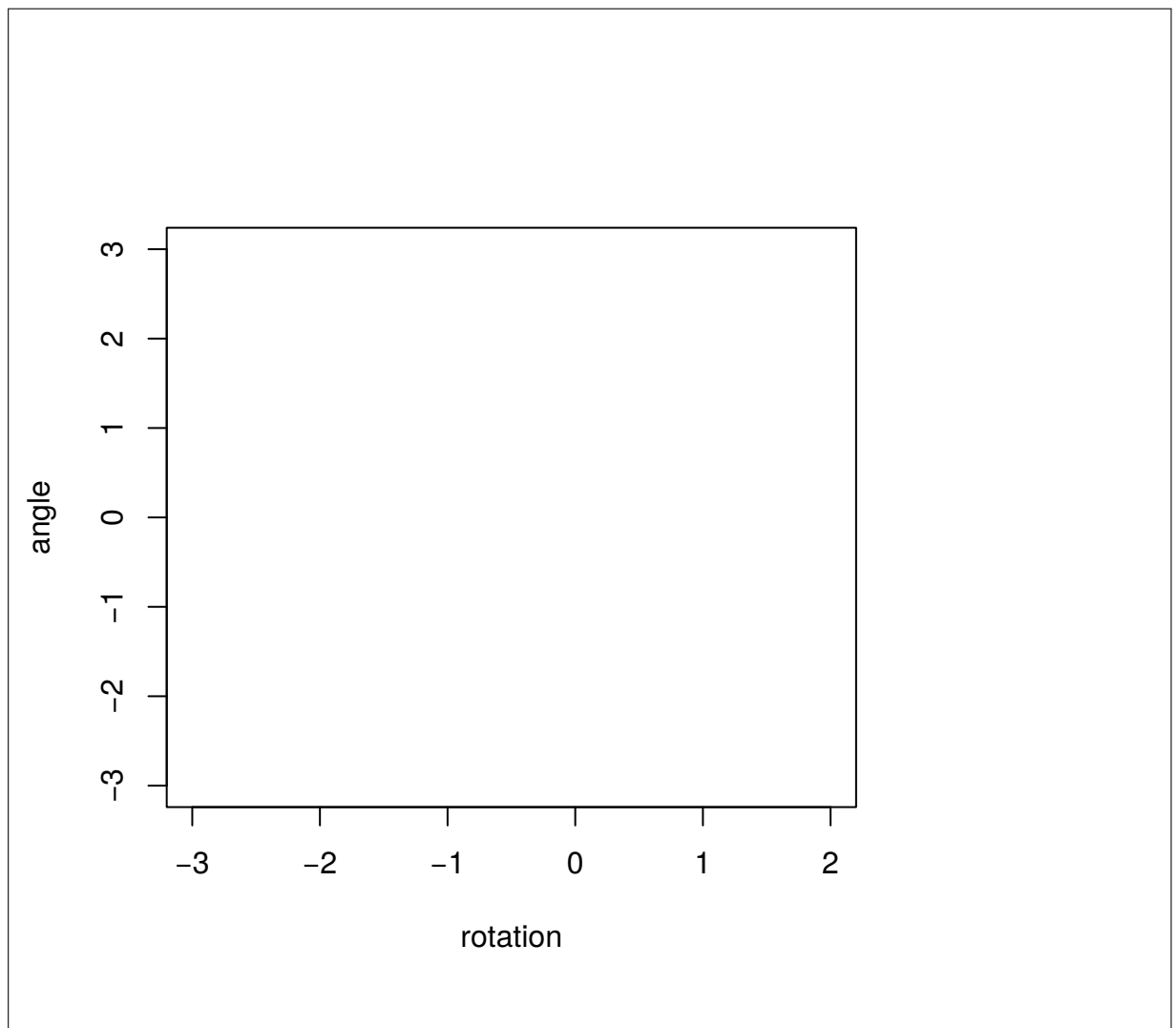
Question	Mark	Comments
1	/ 22	
2	/ 22	
3	/ 17	
4	/ 24	
5	/ 15	
Total		

Question 1. [22 marks] The following data set has measurements of the angle and rotation of a certain industrial process.

```
##      angle rotation
## [1,]      3        2
## [2,]      1       -3
## [3,]     -3       -1
## [4,]      1        2
## [5,]      1        0
## [6,]      2        0
## [7,]      0       -2
## [8,]      0        2
## [9,]     -3        2
## [10,]    -2       -2
```

(a) Complete the scatterplot below.

[4]

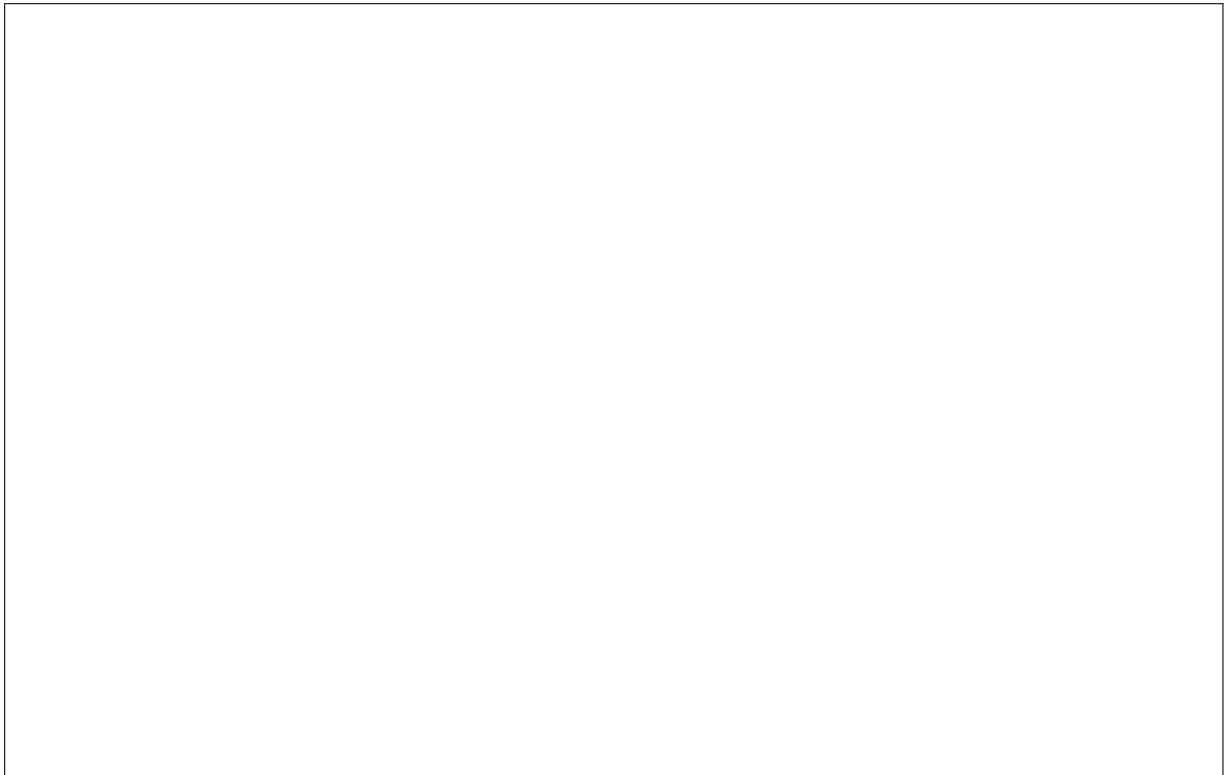


- (b) Let x_1, \dots, x_n and y_1, \dots, y_n be the sample data, each pair x_i, y_i measured on the same item. Write down the formulæ for the sample covariance and the sample correlation. [4]

- (c) Compute the sample covariance for these data. [5]

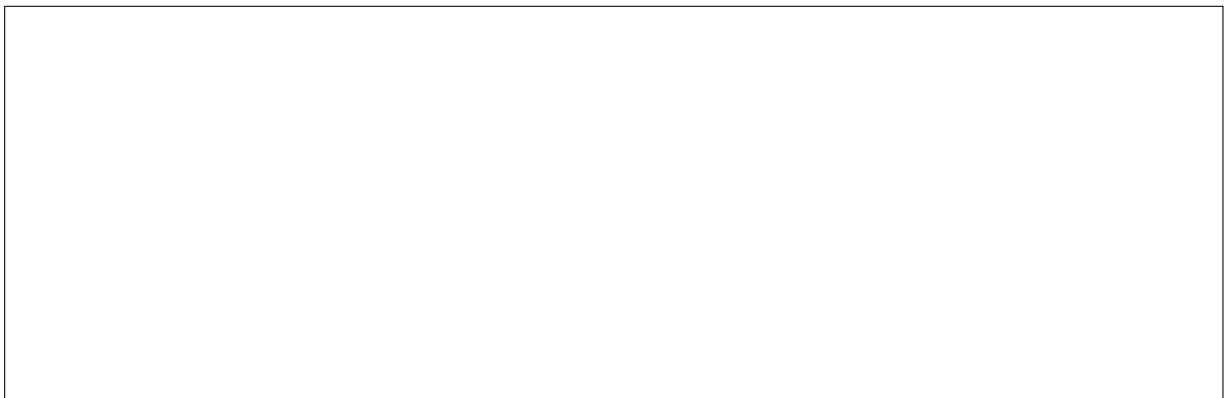
(d) Compute the sample correlation for these data.

[7]



(e) Briefly comment on the relationship between the variables in the data set.

[2]



Question 2. [22 marks]

(a) Compute and give the numerical output of the following commands in R:

[12]

Command	Output
<code>dbinom(x=5,size=8,prob=0.3)</code>	
<code>dpois(x=5,lambda=5)</code>	
<code>dexp(x=1,rate=4)</code>	
<code>pexp(q=1,rate=3)</code>	

(b) Consider the following random uniform(0, 1) numbers:

```
u
## [1] 0.5858 0.0089 0.2937 0.2774 0.8136
```

Give the output of the following R command: `qexp(p=u,rate=2)`. Recall that for an exponential random variable, $F_X(x) = \Pr(X \leq x) = 1 - e^{-\lambda x}$ for $x \geq 0$ and zero otherwise. [10]

Question 3. [17 marks]

- (a) Write down an expression for a $100(1 - \alpha)\%$ confidence interval for the mean of a data set, briefly explaining what the elements in the expression are. [4]

- (b) The following data are measurements of the density of mineral samples in grammes per cubic centimetre:

6, 17, 4, 10, 19, 7, 8, 7, 9, 11

- (i) Define and compute the sample mean \bar{x} and sample variance s^2 for these data. [6]

- (ii) Give a 95% confidence interval for the mean density of mineral samples. Use $\sigma^2 = s^2$. [7]

Question 4. [24 marks]

- (a) Let X and Y be random variables. Write down the formulæ for the covariance $\text{Cov}(X, Y)$ and correlation $\text{Corr}(X, Y)$.

[4]

- (b) Consider two independent standard normal random variables Z_1 and Z_2 . Let $Y = Z_1 + Z_2$ and $W = Z_1 - Z_2$.

- (i) Determine the distributions of Y and of W .

[8]

- (ii) Compute the covariance $\text{Cov}(W, Y)$ and the correlation $\text{Corr}(W, Y)$. [8]

- (iii) Are the random variables W and Y uncorrelated? Are they independent? [4]

Question 5. [15 marks] The stadium expenses on game day of fans of two football teams are to be compared. The following table shows expenses of samples of fans of two teams, collected on a match day between the teams. The table also shows sample summaries for each sample: mean, variance and standard deviation (s.d.).

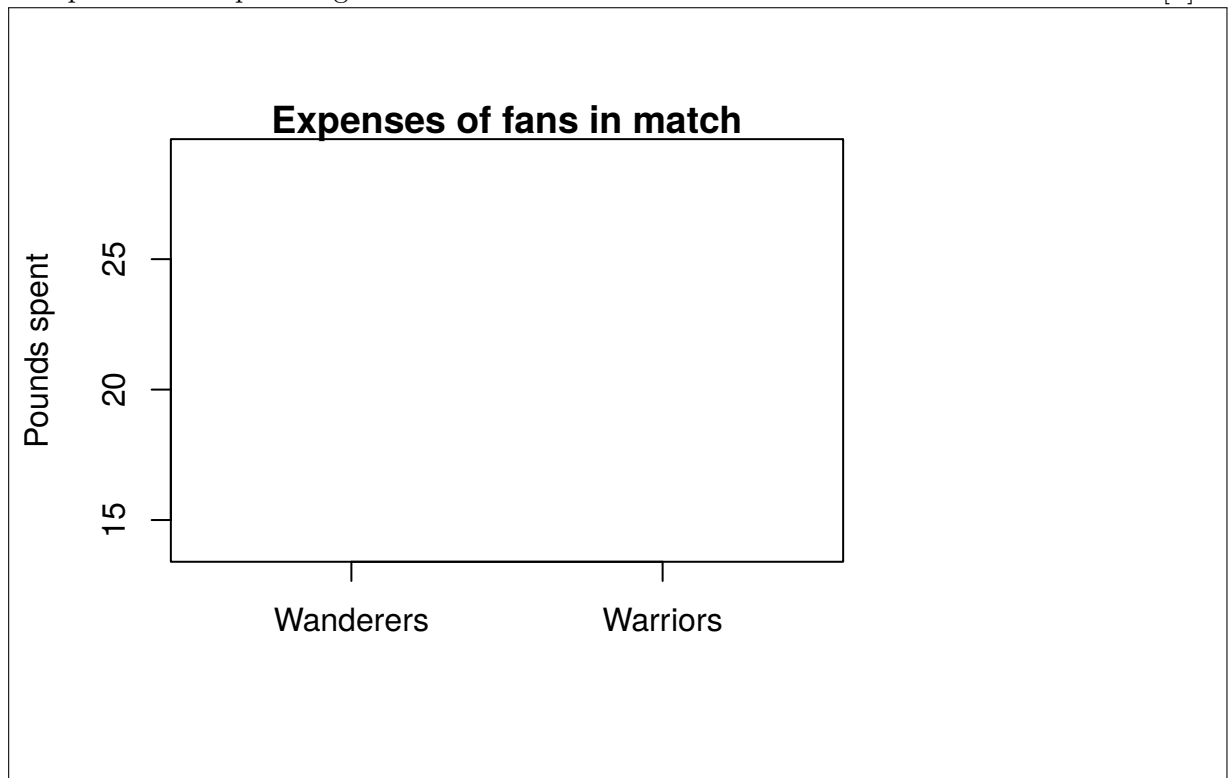
Team	Expenses (£)	Mean	Variance	s.d.
Wanderers	17, 20, 20, 26, 17, 23, 29, 20, 23, 20	21.5	14.5	3.808
Warriors	23, 20, 20, 23, 20, 17, 23, 14, 17, 23	20	10	3.162

(a) Compute the five-number summary for each of the teams.

[8]

(b) Complete the boxplot diagram.

[3]



(c) Compare the distributions of expenses between the two teams using the standard hierarchy of description: location, scale and tails.

[4]

Blank area for student response.

End of Paper.