

Main Examination period 2019

MTH6156 / MTH6156P: Financial Mathematics III

Duration: 2 hours

Student number

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Make and model of calculator used _____

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more paper, ask an invigilator for an additional booklet and attach it to this paper at the end of the exam.

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| You should attempt ALL questions. Marks available are shown next to the questions. |
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This is an OPEN BOOK exam

permitted:

any printed material, e.g., books, any handwritten notes, photocopies of any kind

prohibited:

using communication devices, e.g., laptops or mobile phones, sharing material with other students.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: L. Wunderlich, R. J. Harris

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Do not write on it.

| Question | Mark | Comments |
|----------|------|----------|
| 1 | / 4 | |
| 2 | / 20 | |
| 3 | / 22 | |
| 4 | / 20 | |
| 5 | / 18 | |
| 6 | / 16 | |
| Total | | |

Question 1. [4 marks] Raj Rajaratnam, the head of the Galleon Group, was arrested by the FBI in 2009 for insider trading. The investigations showed that he received confidential information from senior executives of several companies. He used this information for the investment strategy of his company's hedge fund, which manages over \$7 billion. Under the assumption of the semi-strong market hypothesis, is it possible that his investment strategy consistently yields higher profit than the market, or was it simply luck?

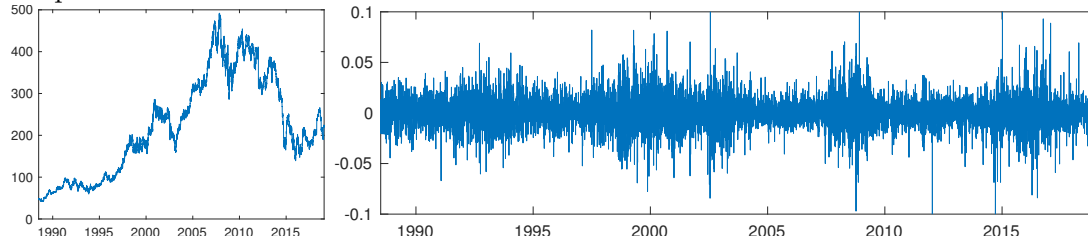
- A. Market efficiency was developed under the assumption that insider trading is not possible. Therefore we cannot decide this question.
- B. The semi-strong form of market efficiency states that investments based on public information cannot consistently beat the market. His investment was based on private information, so the higher profit is not ruled out by the semi-strong form.
- C. The semi-strong form automatically implies the strong form, which says that no investment can ever consistently beat the market. Therefore his investment was purely lucky.

Write here the letter that you select:

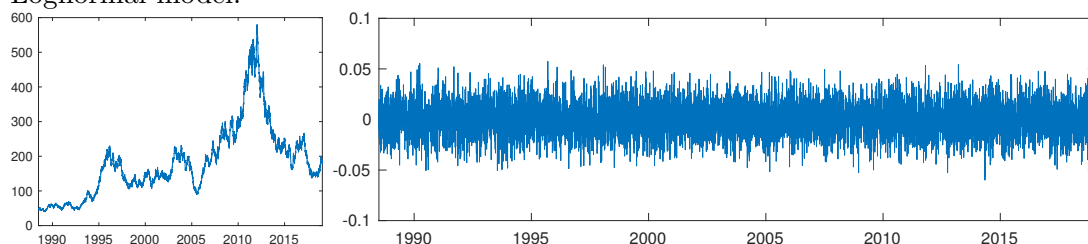
[4]

Question 2. [20 marks] Consider the following plot of stock prices (left) and daily returns (right). We compare the empirical data (top) to one realisation of a lognormal model (bottom).

Empirical data:



Lognormal model:



(a) Which statement is true, when comparing the empirical data to the model data?

- A. The lognormal model does not show volatility clustering, as subsequent returns are modelled to be independent of each other.
- B. Basic market mechanisms guarantee that the market behaves as the lognormal model predicts. The fact that the empirical data look different is due to poor data quality.
- C. Both the model and the empirical data show volatility clustering, e.g., when the stock price is large in the period around 2010.

Write here the letter that you select:

[4]

(b) Assume the weak form of market efficiency. Which statement is correct?

- A. As long as subsequent returns are not correlated, they may still be dependent on each other and in line with market efficiency.
- B. The empirical data shows a clear dependency of subsequent returns. This contradicts the weak form of market efficiency.
- C. For both the empirical data and the log-normal model, subsequent stock prices are correlated. Only due to trading costs, we cannot exploit this correlation.

Write here the letter that you select:

[4]

- (c) Which comment on the quality of the lognormal model is correct?
- A. The lognormal model almost perfectly reflects market behaviour. Therefore there is no use considering any other models.
 - B. Future stock prices are random and therefore cannot be predicted. Thus the lognormal model is useless in practice.
 - C. The model is useful in practice but carries some risks. For example, it underestimates the probability of large losses. Thus the overall risk is underestimated.

Write here the letter that you select: [4]

- (d) Besides the lognormal model shown above, stochastic volatility models (e.g., the autoregressive model) were also considered in the lectures. Which statement is correct?
- A. Stochastic volatility models can improve the lognormal model, as they reflect the fact that subsequent stock prices are highly dependent.
 - B. Stochastic volatility models can improve the model, as they can show volatility clusters.
 - C. Stochastic volatility models are easier to use compared to the lognormal model, because the model parameters are easier to fit.

Write here the letter that you select: [4]

- (e) The lognormal model assumes subsequent daily log-returns X_t to be independent and identically distributed: $X_t \sim \mathcal{N}(\mu, \sigma^2)$. Under this assumption, what are the mean value and the variance of the weekly log-returns $\sum_{i=1}^5 X_i$, in terms of σ and μ ? (We assume five trading days per week.) Briefly justify your answer. [4]

Write your solutions here

Question 3. [22 marks] You are responsible for a portfolio that consists of £60 000 invested in a risk-free asset with zero interest rate, 3 000 stocks of company A and 1 000 stocks of company B. The stock-prices are shown in the following figure, where we see that they are currently $S_0^A = £26.00$ for company A and $S_0^B = £18.50$ for company B.

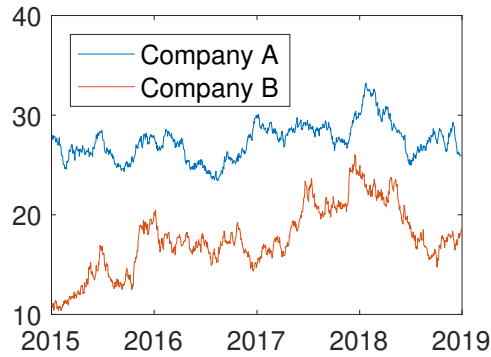


Figure: Historic stock price of companies A and B.

(a) What is the portfolio's current value?

[4]

Write your solutions here

- (b) The portfolio should always have at least 25% of its value invested in the risk-free asset. Is this criterion fulfilled at the current time? Justify your answer by computing the current proportion invested in the risk-free asset. [4]

Write your solutions here

- (c) Assume for this sub-question that the stock price of company B does not change and that the condition of part (b) applies. The portfolio will need rebalancing when the stock price of company A:
- (i) gets too large;
 - (ii) remains the same;
 - (iii) gets too low.

Choose one of the above without justification.

[2]

- (d) Derive the variance of the return of the portfolio in terms of the variances of the returns of the single assets $\text{Var}(R^A)$, $\text{Var}(R^B)$ and their covariance $\text{Cov}(R^A, R^B)$.

You may round to two digits after the decimal point. Carefully justify all steps in your derivation.

Hint: You will need to correctly account for the different proportions of the two assets. [12]

Write your solutions here

Question 4. [20 marks] Consider a market where all assumptions of the CAPM with a return of the risk-free asset of $\mu_0 = 2\%$ are satisfied. We observe that the expected return of the market portfolio is $\mu_M = \mathbb{E}(R^M) = 22\%$ and its standard deviation is $\sigma_M = \sqrt{\text{Var}(R^M)} = 20\%$.

- (a) Does a portfolio with expected return of 30% and a standard deviation of 20% exist? Briefly justify your answer. [4]

Write your solutions here

You would like to create an efficient portfolio with less risk, more specifically with a standard deviation of $\sigma_P = \sqrt{\text{Var}(R^P)} = 10\%$.

- (b) What is the expected return of this efficient portfolio? Carefully justify all steps in your derivation.

Hint: It could be helpful to first derive β (the Beta of the portfolio), and then to express the expected return in terms of β and μ_M . [12]

- (c) How is this portfolio constructed? [4]

Write your solutions here

Question 5. [18 marks] You have investments in two rival companies of a well-performing market. You know that one of the companies will gain a large contract, so you review your investment.

You know that the company which wins the contract yields a return of 10%, while the other company yields a loss of 5% as expectations are not met. With your knowledge of the market, you believe that company A has a 40% chance to win the contract, in the other case (with the remaining probability of 60%), company B gets the contract.

In the following, we assume that your estimated probabilities are correct.

| winner | return company A | return company B | likelihood |
|-----------|------------------|------------------|------------|
| Company A | +10% | -5% | 40% |
| Company B | -5% | +10% | 60% |

- (a) State the distribution of the random variables for each company's return individually and the joint distribution.

[6]

Write your solutions here

- (b) Compute your portfolio's expected return if you have invested equally in both companies.

[4]

Write your solutions here

(c) Construct a portfolio with minimal variance.

[4]

Write your solutions here

- (d) Without short selling, or the possibility to lend money, construct a portfolio which maximises the expected return.

[4]

Write your solutions here

Question 6. [16 marks] You have the choice between two lotteries, L_1 and L_2 :

$$L_1 = \begin{cases} -2, & \text{with probability } 1/2, \\ 2, & \text{with probability } 1/2. \end{cases}$$

$$L_2 = \begin{cases} -2, & \text{with probability } 1/4, \\ 0, & \text{with probability } 1/2, \\ 2, & \text{with probability } 1/4. \end{cases}$$

- (a) You are a risk-seeking investor. Which lottery maximises your expected utility? Briefly explain your reasoning.

[8]

Write your solutions here

(b) Which of the following functions are valid utility functions in accordance with you being risk-seeking? Explain your answer.

A. $u(x) = x$,

B. $u(x) = e^x - 1$,

C. $u(x) = x^2$.

[8]

Write your solutions here

Extra space to continue your answers.
Please mark clearly, which question your answer belongs to.

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Please mark clearly, which question your answer belongs to.

End of Paper.