

Main Examination period 2018

**MTH6156 / MTH6156P: Financial Mathematics III**

**Duration: 2 hours**

Student number 

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Desk number 

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Make and model of calculator used \_\_\_\_\_

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more paper, ask an invigilator for an additional booklet and attach it to this paper at the end of the exam.

**You should attempt ALL questions. Marks available are shown next to the questions.**

This is an **OPEN BOOK** exam

**permitted:**

any printed material, e.g. books

any handwritten notes

photocopies of any kind

**prohibited:**

using communication devices, e.g. laptops or mobile phones

sharing material with other students.

**Multiple choice:** Marks are given for correct answers, no negative marks are given.

**Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.**

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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**Exam papers must not be removed from the examination room.**

**Examiners: K. Glau, O. Bandtlow**

This page is for marking purposes only.  
**Do not write on it.**

Question	Mark	Comments
1	/ 4	
2	/ 4	
3	/ 12	
4	/ 20	
5	/ 28	
6	/ 20	
7	/ 12	
<b>Total</b>		

**Question 1. [4 marks]**

We read the following news: "A new company called 'FutureGains' has launched a new fund. The company uses market data and a new algorithm to find arbitrage possibilities."

The investors in FutureGains trust the strategy. Which form of the efficient market hypothesis do they believe is wrong?

- A. The weak-form hypothesis of market efficiency.
- B. The half-baked form hypothesis of market efficiency.
- C. The none of the hypotheses of market efficiency.
- D. The hypotheses of poor markets.

Write here the letter that you select:

[4]

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**Write your solutions here**

**Question 2. [4 marks]**

What is the relation of the weak efficient market hypothesis to stochastic models of asset price evolution?

- A. There is no such relation. The weak EMH is an economic illusion while stochastic models are mathematically correct.
- B. The weak EMH contradicts the stochastic model of stock prices: Asset prices reflect insider knowledge and hence a stochastic model is useless.
- C. The weak EMH is in line with stochastically modelling asset price evolution: Historical prices do not allow to predict future stock prices with certainty. The same holds in a stochastic model.
- D. The weak EMH is in line with stochastically modelling asset price evolution: Squared daily returns of successive days are positive so the returns are not independent and the weak EMH states the dependence of daily returns.

Write here the letter that you select:

[4]

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**Write your solutions here**

**Question 3. [12 marks]**

Inspect the plots of data of daily stock returns in Figure 1.

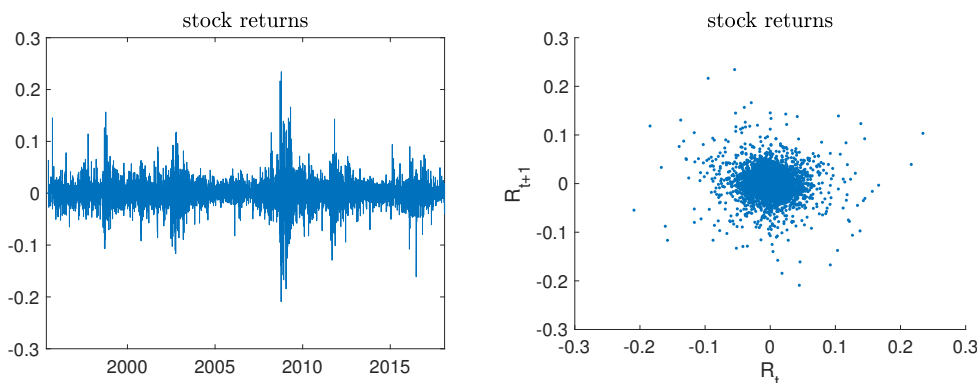


Figure 1: Representations of financial data.

PART I: What is a correct statement related to the left graph?

- A. In the period around 2008 the largest negative spike of returns is observed. This largest daily loss is called the volatility cluster.
- B. The data clearly exhibits volatility clustering. The cluster is at zero because returns are typically small.
- C. We observe volatility clustering. Namely, large absolute returns are frequently observed to follow each other, and we also observe periods with low absolute returns.

Write here the letter that you select for PART I:

[4]

PART II: What is a correct statement related to the right graph?

- D. The largest daily loss observed in the data is analyzed in more detail in the right graph, where we can see that it is not predicted from a model.
- E. The data clearly exhibits volatility clustering. The cluster is seen in the right graph, it is the center of the shotgun pattern.
- F. The graph shows the daily returns versus the daily returns of one-day ahead. The shotgun pattern is an indication that successive daily returns are nearly uncorrelated.

Write here the letter that you select for PART II:

[4]

PART III: What is a correct statement?

- G. Volatility clustering means that large successive daily returns are likely to follow each other. This contradicts the hypothesis that successive daily returns are uncorrelated.
- H. Volatility clustering means that periods of high market activity in terms of buy-and-sell follow idle periods. This influences the prices and hence the stock returns follow a clock-like pattern during the day as seen in the right graph.
- J. Successive daily returns can be uncorrelated while their squared values are correlated and this is often observed.

Write here the letter that you select for PART III:

[4]

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Write your solutions here

Question 4. [20 marks]

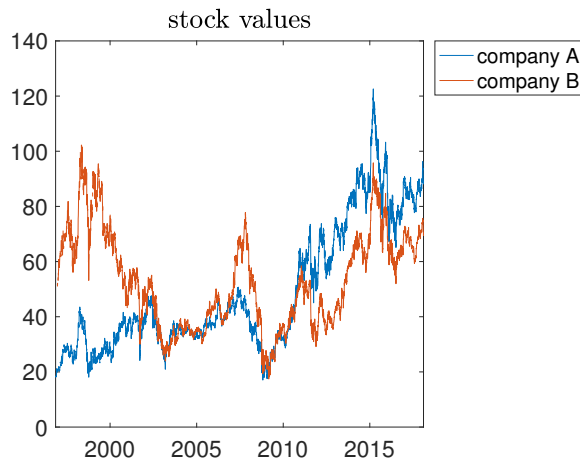
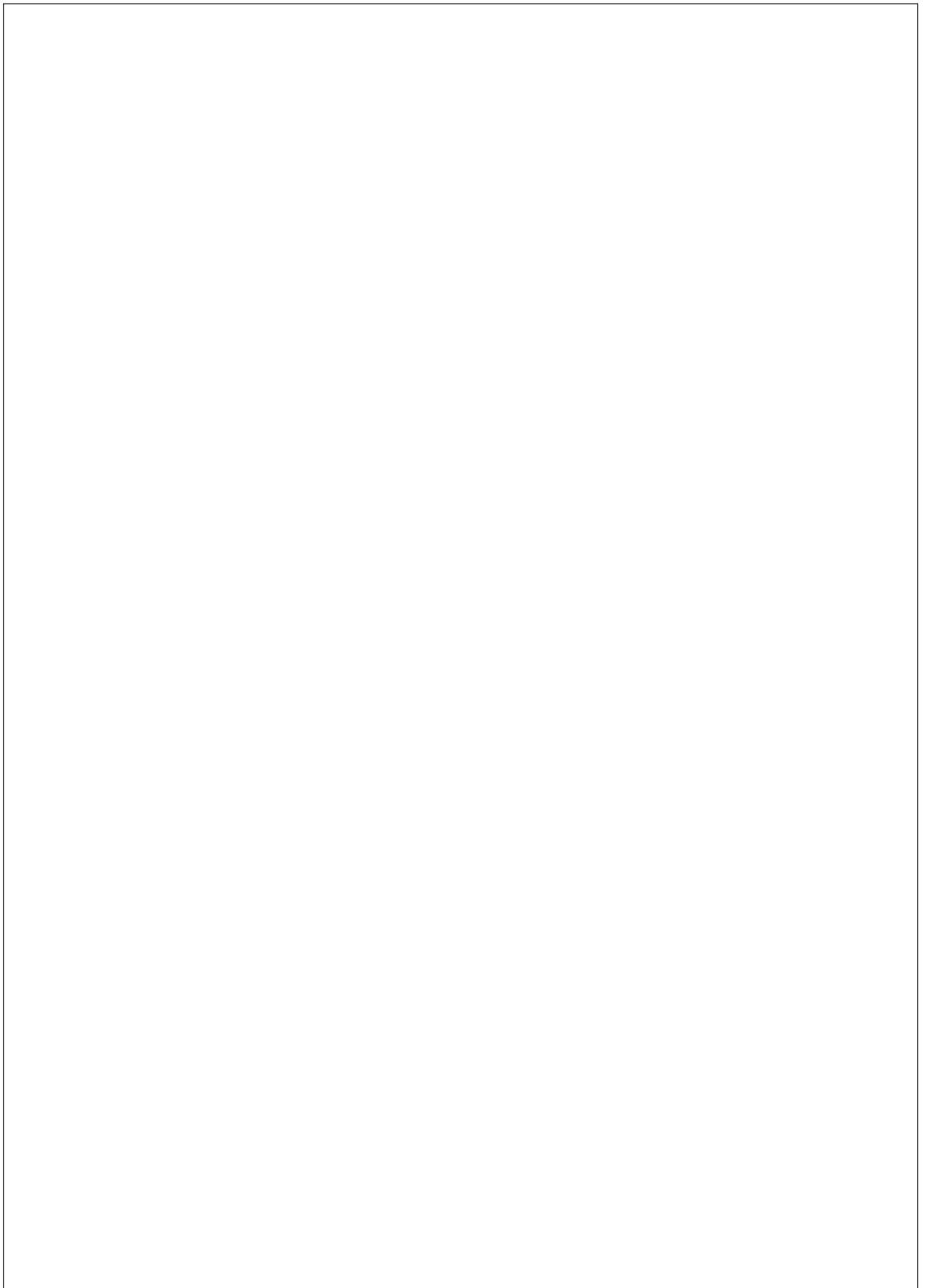


Figure 2: Representation of financial data.

You are responsible for a portfolio that consists of 3,000 stocks of company A and 1,000 stocks of company B. The data of daily stock returns are depicted in Figure 2.

- I. (a) What is the portfolio's value on the last day reported for which the stock value of company A was 89.1 and of company B was 70.65?
- (b) At that time, what is the proportion of the portfolio invested in the stocks of company A and in the stocks of company B?

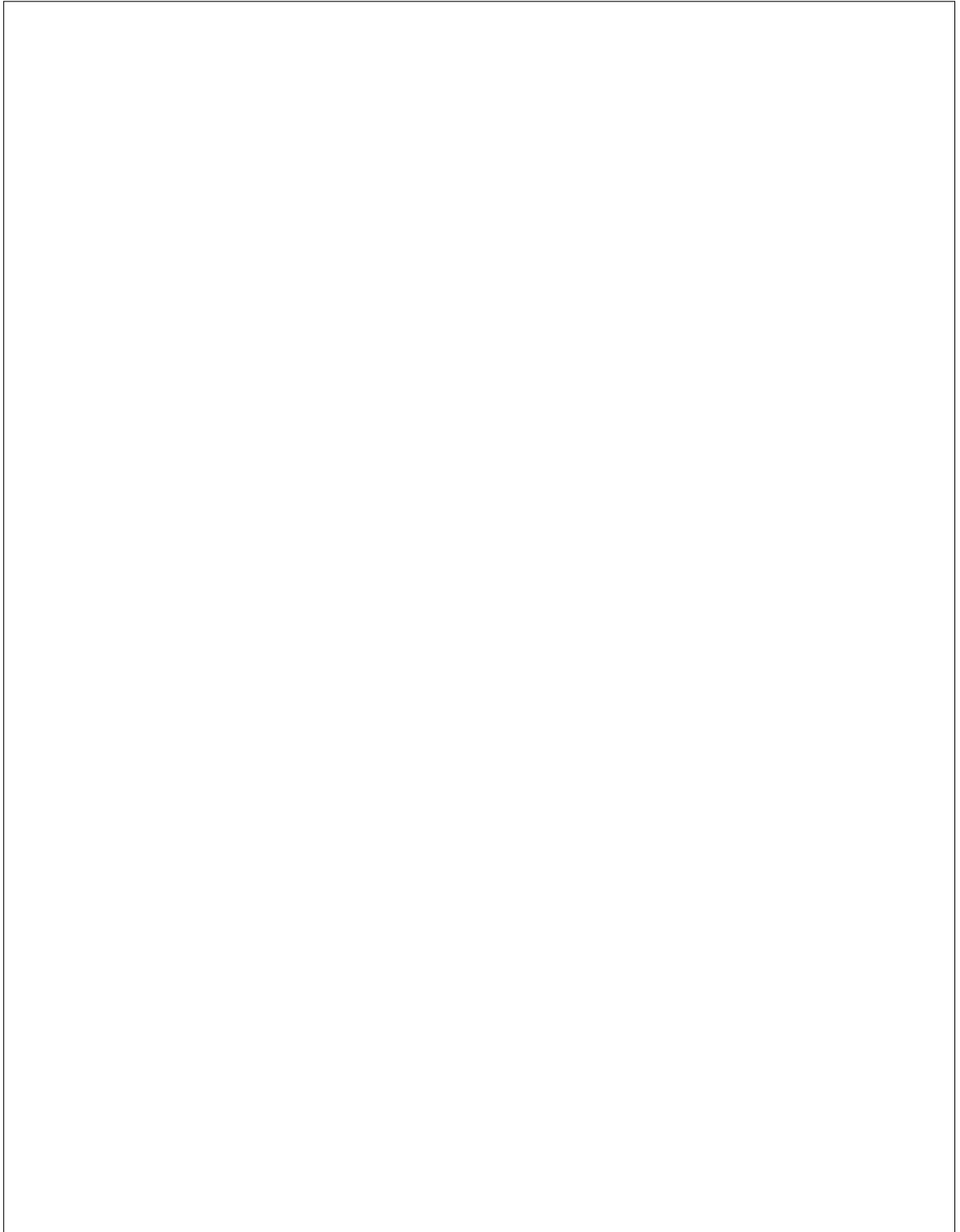
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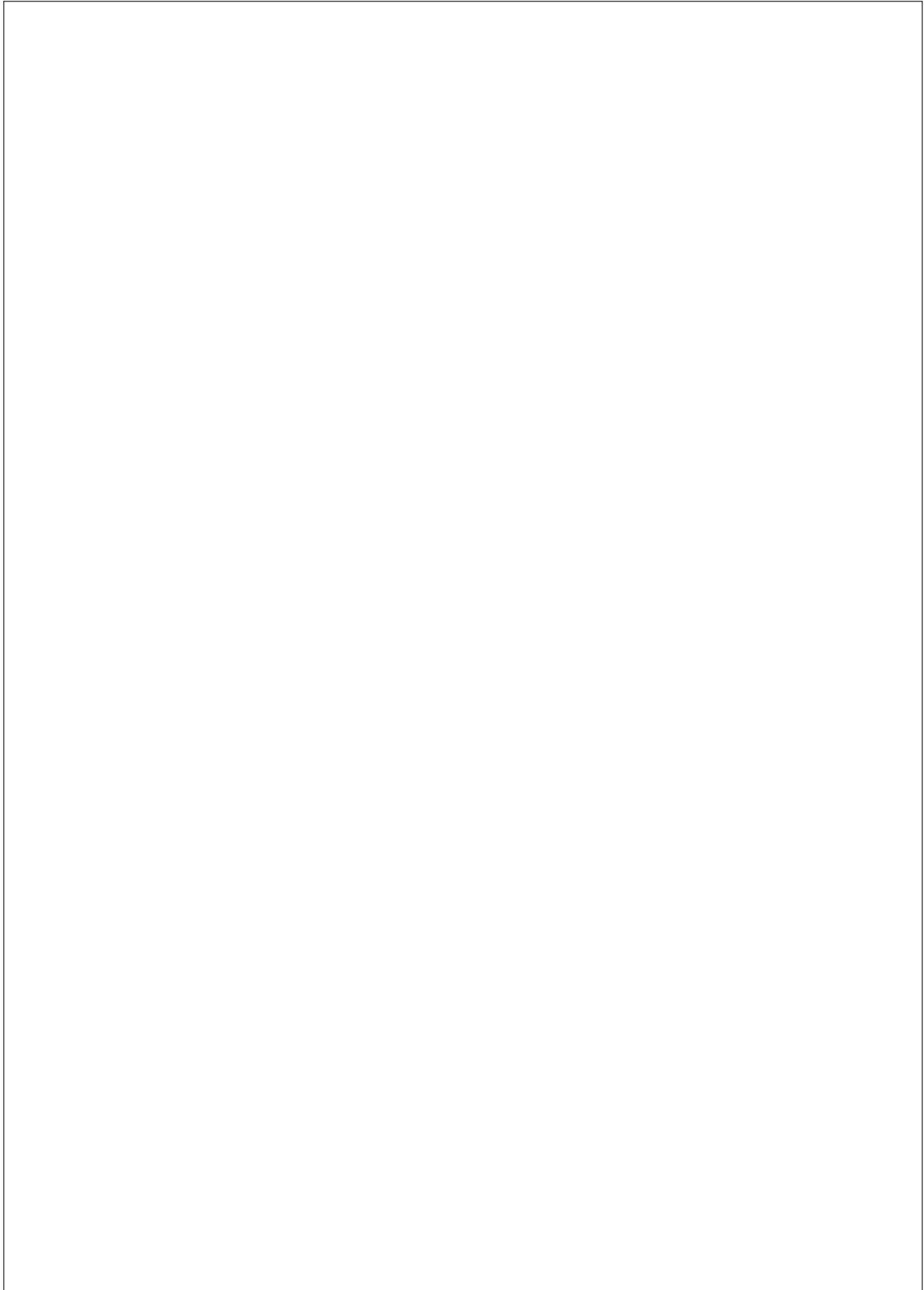


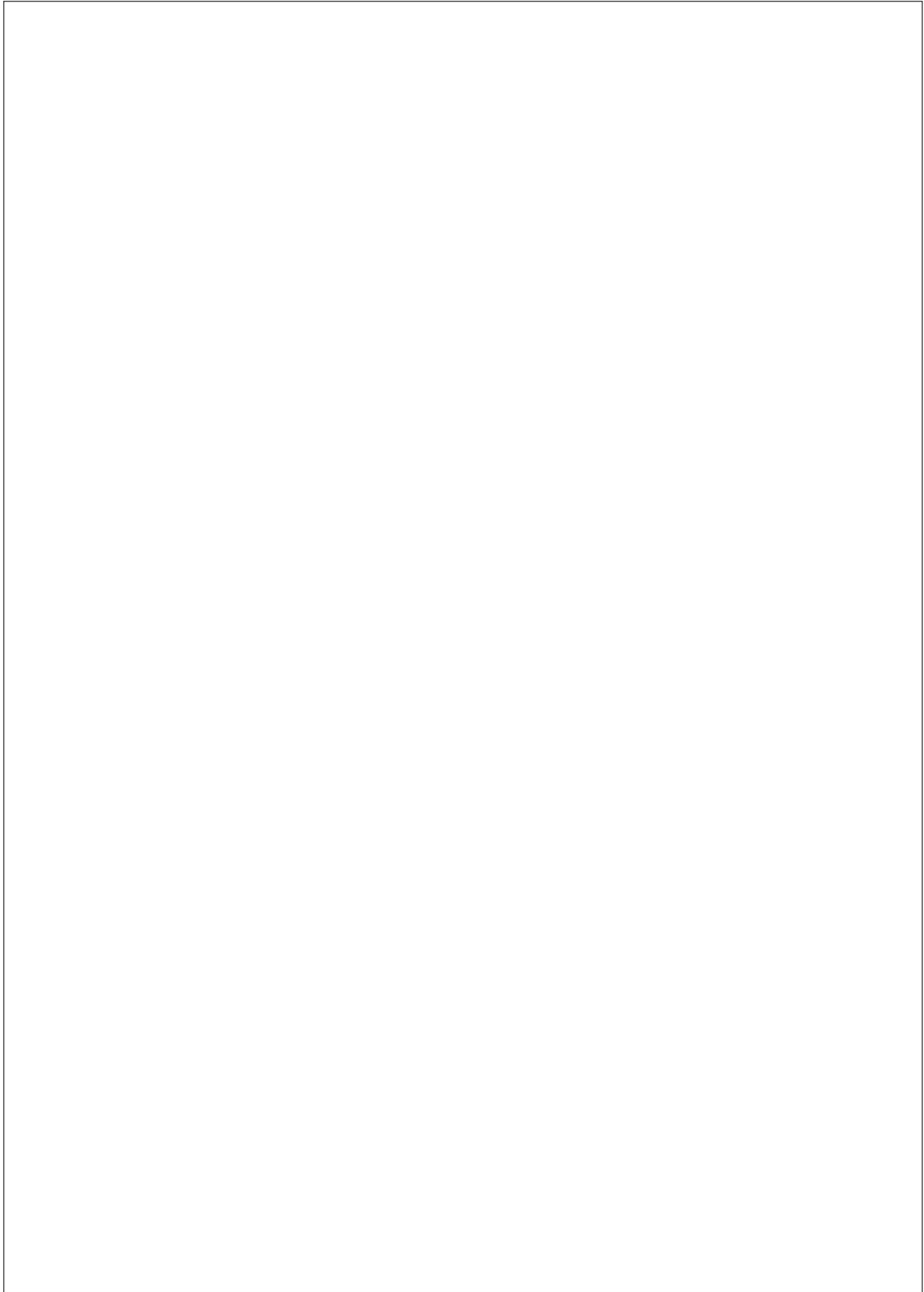
- II. Derive the variance of the return of a portfolio of two assets using the variances of the returns of the single assets and their correlation. (Notice that the portions may differ.) Carefully justify all steps in your derivation.

[12]



Write your solutions here





**Question 5. [28 marks]**

Tottenham Hotspur and Chelsea FC face off in an important final. It has been found in earlier years that the fan shop of the winner’s team makes a large portion of its yearly business in selling fan articles right after that big success. Both teams are very strong, however, based on sophisticated analysis of their performances during the season we believe that Tottenham’s team will win with probability 60%.

winner	return Tottenham’s shop	return Chelsea’s shop	likelihood
Tottenham	+30%	−10%	0.6
Chelsea	−10%	+30%	0.4

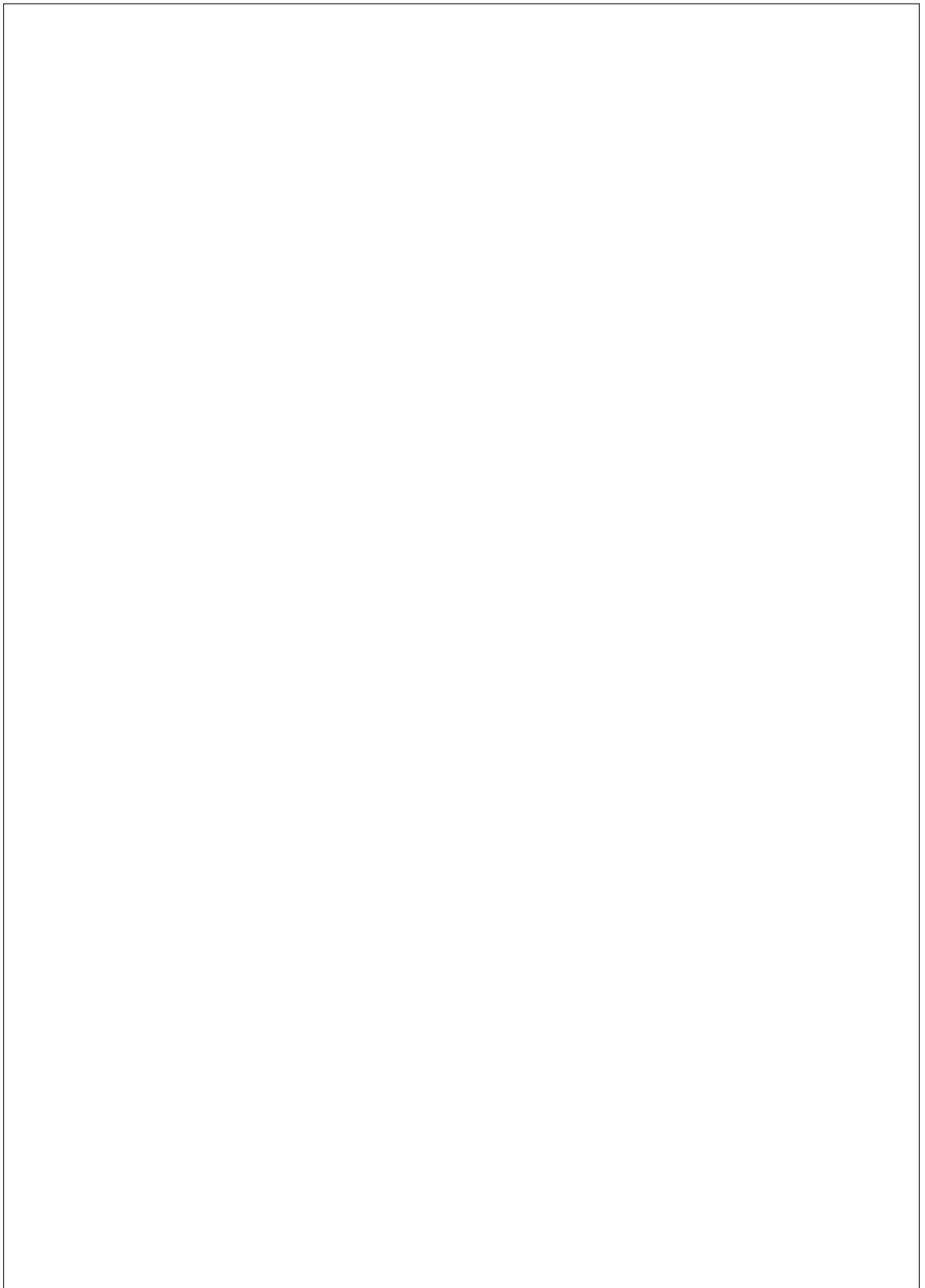
I. Provide a stochastic model for the returns.

[8]

II. Compute the expected returns and the variances of the returns.  
(Show all your working.)

[12]

A large empty rectangular box with a thin black border, intended for the student to show their working for the problem above.



III. What does the correlation  $\rho$  between the returns satisfy?

- D.  $\rho = 0$ .
- E.  $\rho = -1$ .
- F.  $\rho \in (0, 1)$ .
- G.  $\rho \in (-1, 0)$ .

Write here the letter that you select for PART III.:

Briefly justify your answer:

[8]

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Write your solutions here



**Question 6. [20 marks]**

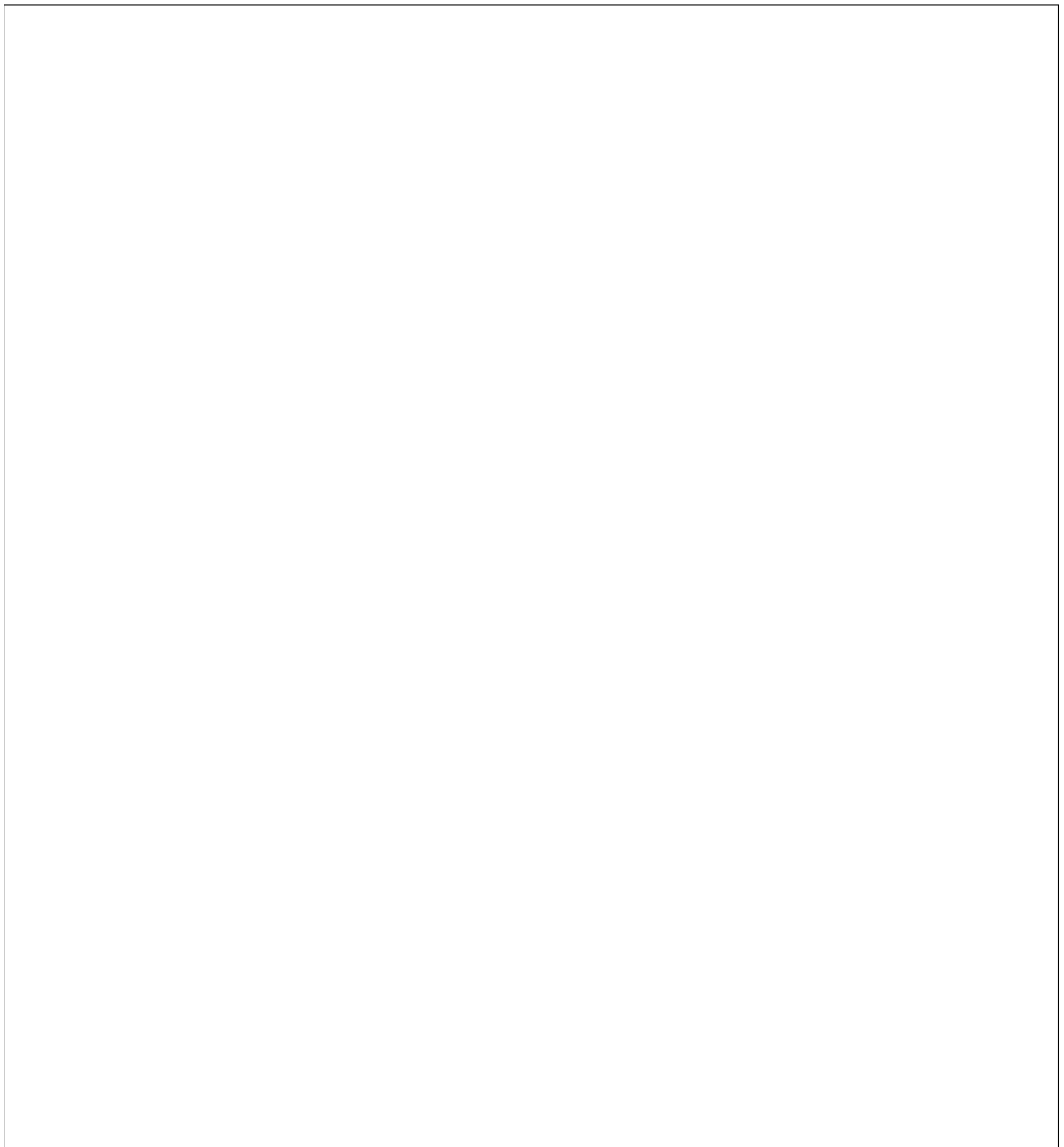
Consider a market where all assumptions of the CAPM with interest rate  $r = 2\%$  are satisfied. We observe that the expected return of the market portfolio is  $\mu_M = E[R^M] = 22\%$  and its standard deviation is  $\sigma_M = \sqrt{\text{Var}(R^M)} = 25\%$ . Consider an efficient portfolio with expected return  $R^P$ .

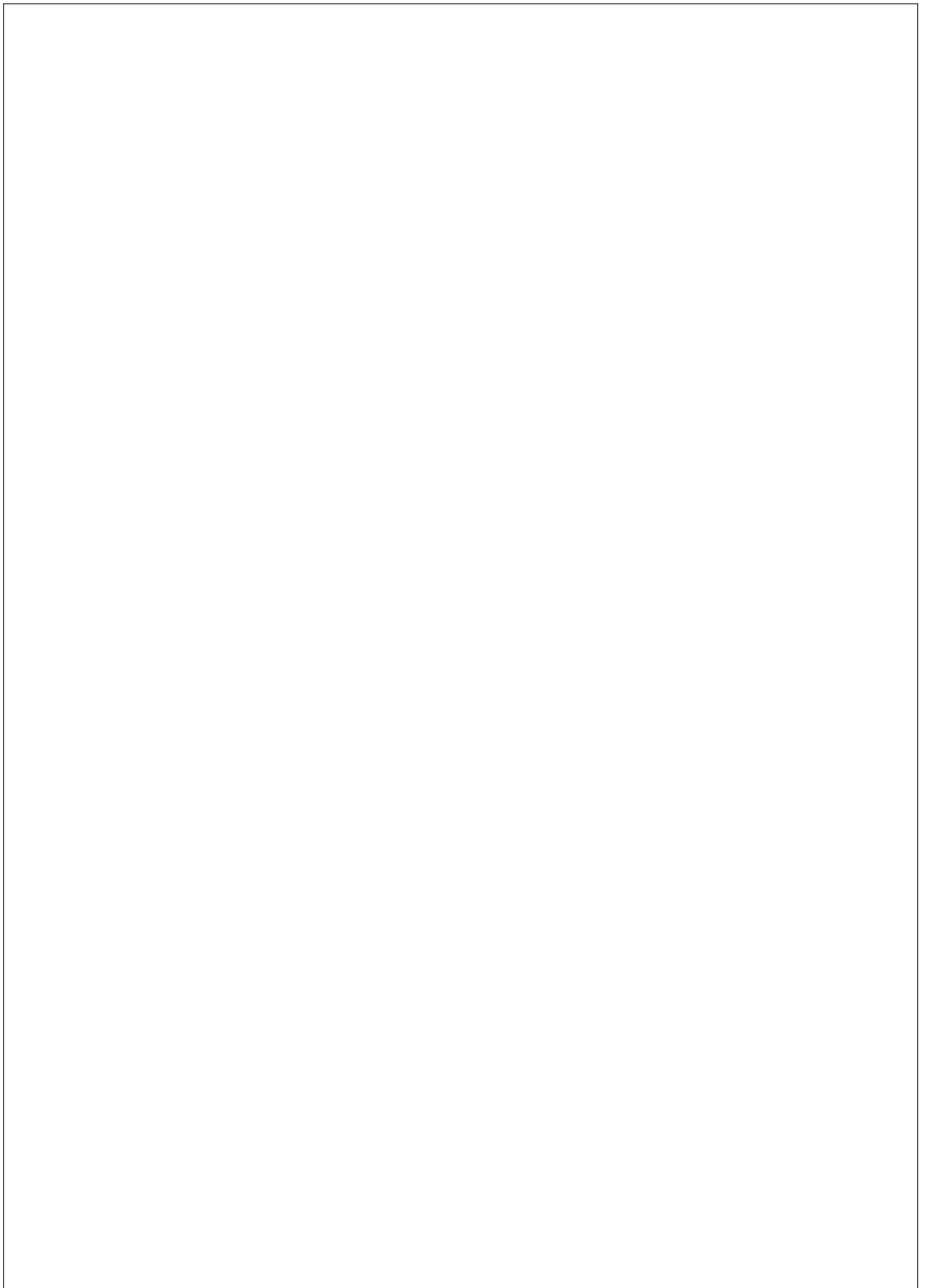
- I. What is the standard deviation of this efficient portfolio, if  $\mu_P = E[R^P] = 12\%$ ?

It could be helpful to first derive  $\beta$  (the Beta of the portfolio), and then to express the standard deviation of the portfolio in terms of  $\beta$  and the standard deviation of the market portfolio.

Carefully justify all steps in your derivation.

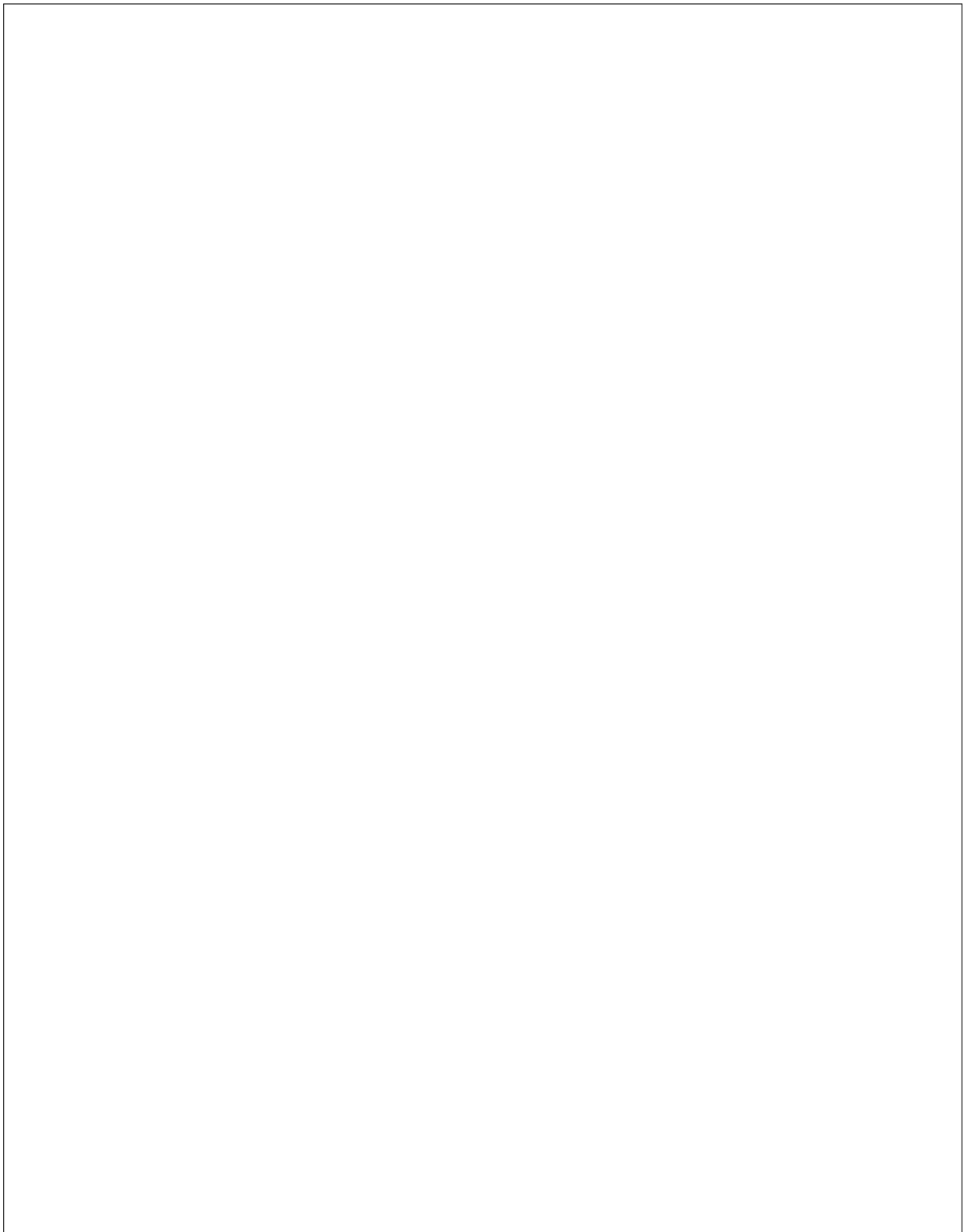
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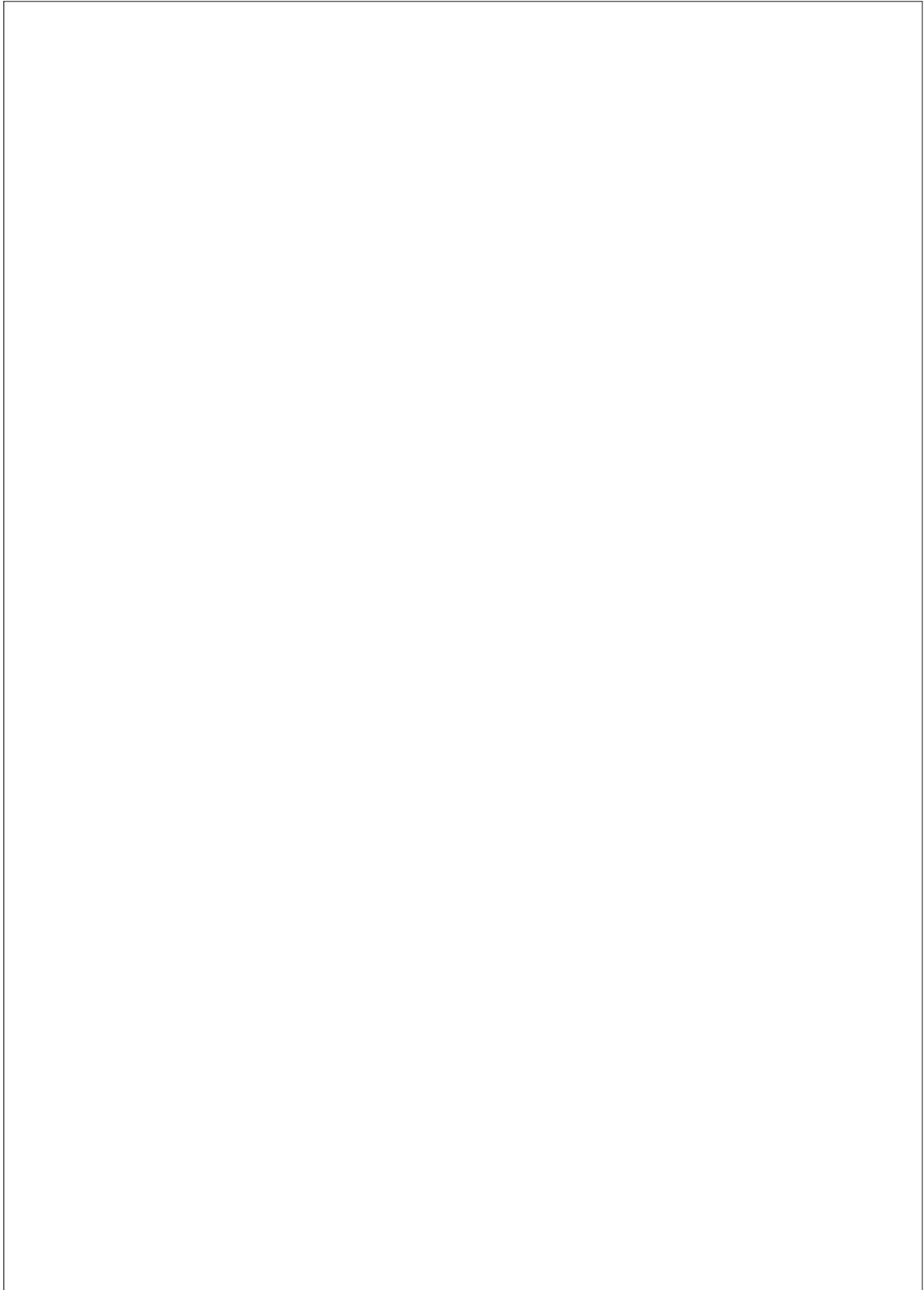




II. What is the composition of the portfolio?

[4]





**Question 7. [12 marks]**

We consider the following two lotteries,  $L_1$  and  $L_2$ :

$$L_1 = \begin{cases} -10, & \text{with probability } \frac{1}{2}, \\ 1, & \text{with probability } \frac{1}{2}. \end{cases}$$

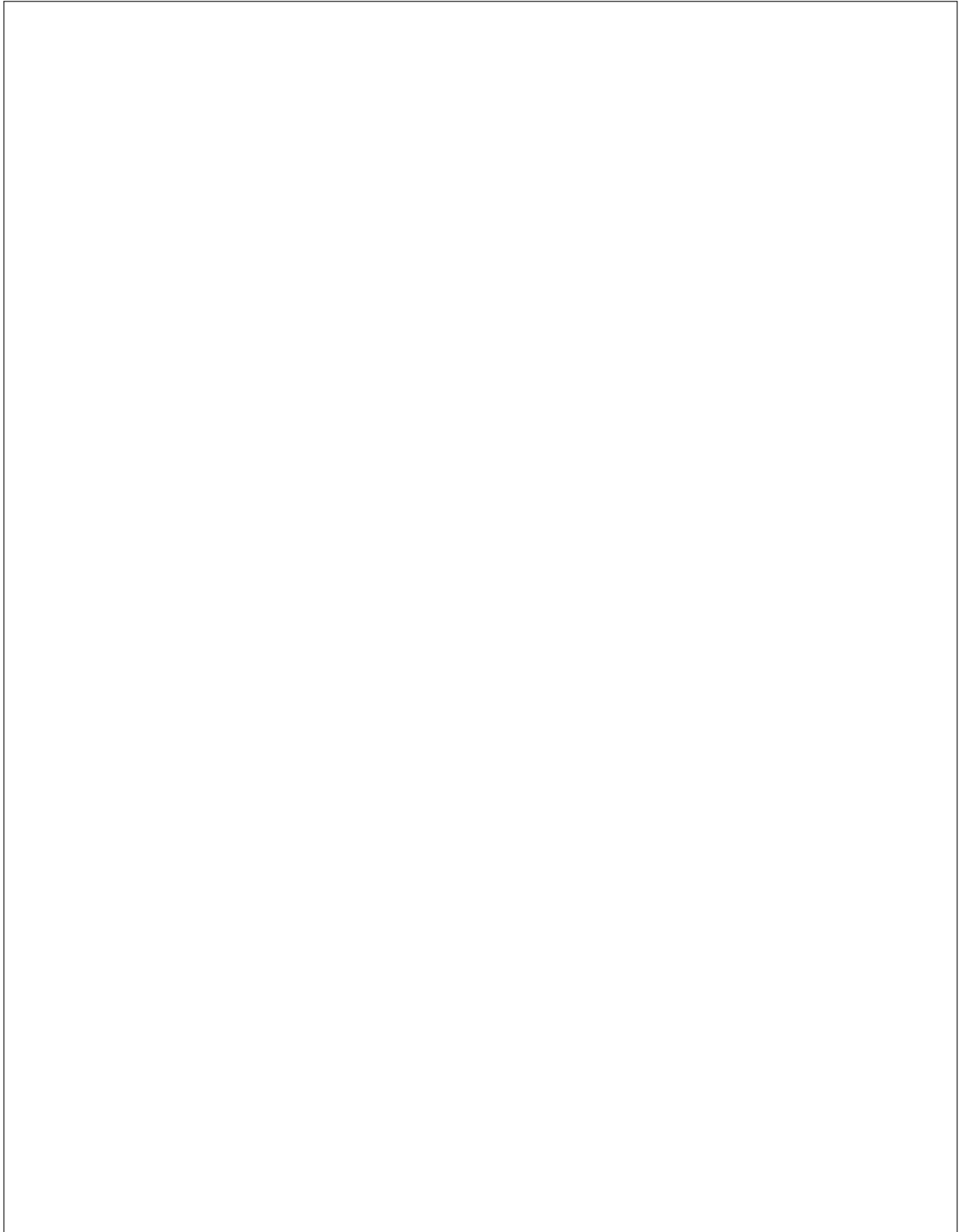
$$L_2 = \begin{cases} -10, & \text{with probability } \frac{1}{4}, \\ 0, & \text{with probability } \frac{1}{2}, \\ 1, & \text{with probability } \frac{1}{4}. \end{cases}$$

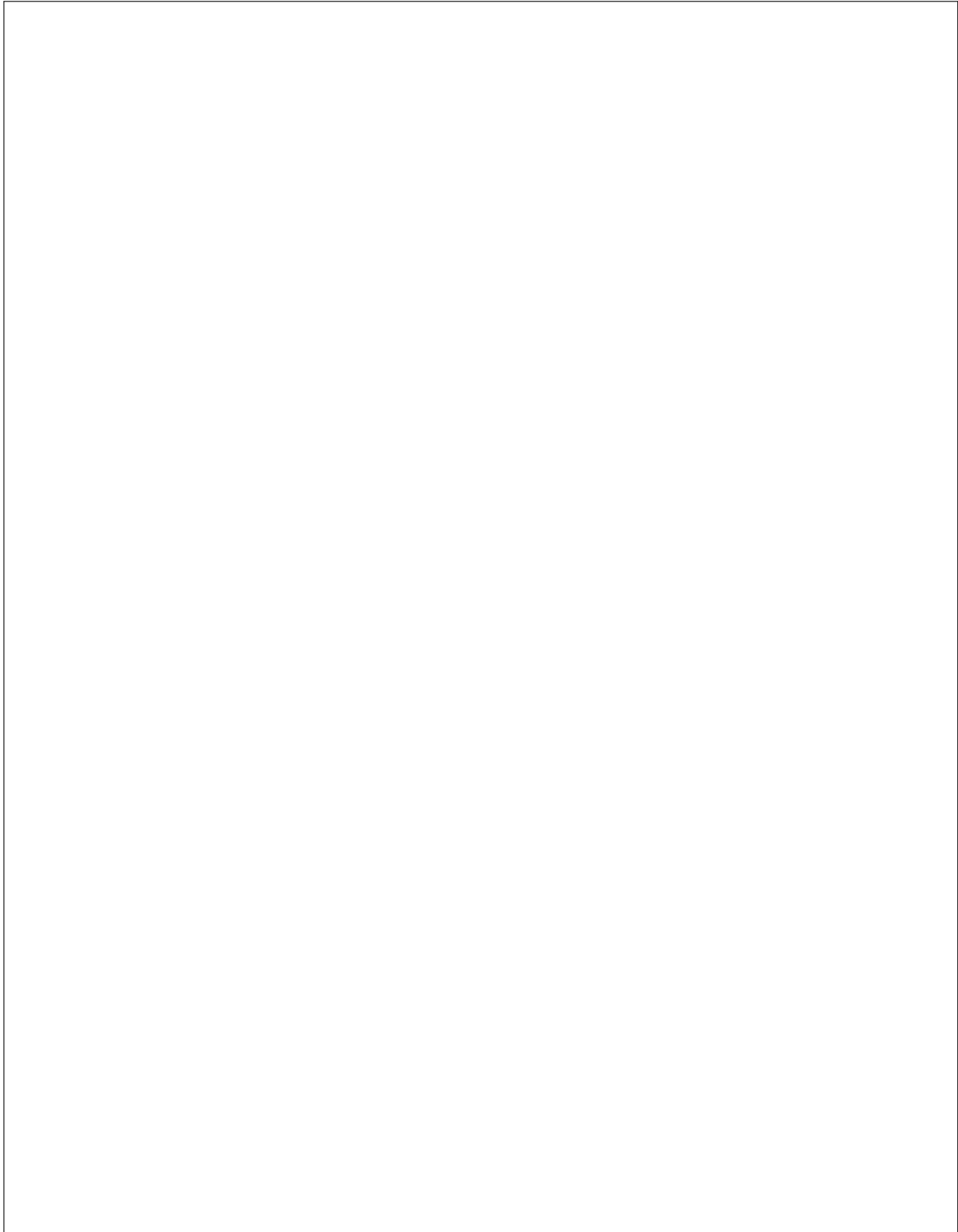
An investor tells us that he/she is risk averse and is maximizing his/her utility choosing lottery  $L_1$  over  $L_2$ . Does he/she tell the truth or does he/she lie?

You may find it useful to include the following steps: Firstly, express the preference in terms of an inequality for the expected utilities. Here you can use a utility function that is not fully specified at this point. Secondly, it could be helpful to express "risk averse" in terms of properties of the associated utility function.

Carefully justify all steps in your derivation.

[12]





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**End of Paper.**