Queen Mary
University of London

Main Examination period 2023 – January – Semester A

MTH6154 / MTH6154P: Financial Mathematics I

Duration: 2 hours

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

Examiners: Dr. D. Stark, Dr. J. Griffin

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Question 1 [23 marks].

(a) Suppose you put £1500 in a bank account with nominal interest rate 1.2% and make no withdrawals. How much money will be in the account 6 years later if the interest is compounded monthly? State your answer to the nearest pence. **[3]**

(b) Suppose a bank account has a nominal interest rate of 4.4% compounded semi-annually. Find the effective interest rate r_{eff} to three significant figures. **[3]**

(c) Suppose that for time $t \geq 0$ the instantaneous interest rate of a bank account is given by

$$r(t) = 0.016 + 0.01te^{-t^2}.$$

(i) Determine the yield curve $\overline{r}(t)$. **[4]**

(ii) Determine $\lim\limits_{t\to\infty} \overline{r}(t)$. **[3]**

(d) Suppose that Bank A offers deposits and loans continuously compounded with discount factor $D_A(t)$ and that Bank B offers deposits and loans continuously compounded with discount factor $D_B(t)$. Moreover, suppose that

$$D_A(1)D_A(2) > D_B(3).$$

Show that an arbitrage opportunity exists. **[10]**

Question 2 [8 marks]. Consider the cash flow $(a_1, a_2, a_3) = (-2, 1, -1)$, where a_i is the payment at the beginning of year i for $i = 0, 1, 2$.

(a) Show that this cash flow does not have an Internal Rate of Return. **[5]**

(b) Why is this cash flow not subject to the theorem proved in lectures about the existence of an Internal Rate of Return r satisfying $-1 < r < \infty$. **[3]**

Question 3 [9 marks]. A 2-year bond has face value £700,000 semi-annual coupons at rate 3% per annum, and is redeemable at half par. The current rate of interest is 3% compounded continuously.

(a) Determine the coupon and redemption payments in pounds. **[4]**

(b) Determine the no-arbitrage price of the bond to the nearest pound. **[5]**

Question 4 [10 marks]. Consider the three cash flow streams of the form (a_1, a_2, a_3) where a_i is the amount of money in thousands of pounds received at the end of year i for $i = 0, 1, 2$:

$$\mathbf{x} = (2, 2, 2)$$
$$\mathbf{y} = (a, 2, 2)$$
$$\mathbf{z} = (2, 2, a),$$

where $a > 2$. Interest is 3% compounded continuously. Order **x**, **y**, and **z** from smallest effective duration to largest effective duration. Justify your answer. **[10]**

Question 5 [14 marks]. Suppose that in the fixed interest rate model the interest rate compounded yearly has the continuous distribution $R \sim \text{Uniform}(1.3\%, 2.7\%)$.

(a) Determine the probability that £200 accumulates to less than £210 after three years? State your answer as a decimal to three significant figures. **[5]**

(b) Find the expected present value of a payment of £10,000 received five years from now. State your answer to the nearest pound. **[6]**

(c) Find the present value of a payment of £10,000 received five years from now if interest is not random any more, but is compounded yearly at rate $\mathbb{E}(R)$, where $\mathbb{E}$ denotes expected value. State your answer to the nearest pound. **[3]**

Question 6 [8 marks].

(a) Assume that Corner Bank quotes spot rate rate $s_8 = 1.5\%$ and forward rate $f_{8,10} = 1.9\%$. Find the spot rate s_{10}. State your answer as a percentage to three significant figures. **[4]**

(b) Suppose that the price of 100 6-year zero-coupon bonds each paying £1 is £96 and that the price of 120 8-year zero-coupon bonds each paying £1 is £105. Assuming there is no-arbitrage, find the forward rate $f_{6,8}$. State your answer as a percentage to three significant figures. **[4]**

Question 7 [8 marks].

A company issues new shares to fund a new manufacturing plant. Explain the meaning of the Arbitrage Theorem with respect to the price of the new shares. **[8]**

Question 8 [20 marks]. A share price is modelled via a two-period binomial model with initial stock price $S = 250$, up/down multiplication factors $u = 1.2$ and $d = 0.8$, and interest rate 3.2% compounded continuously.

(a) Verify that the no-arbitrage assumption is valid in this model. **[3]**

(b) Find the risk-neutral probabilities of up and down movements in the share price. State your answers to three significant figures. **[4]**

(c) Find the no-arbitrage price of a European call option on the share with strike $K = 200$ and expiry date $T = 2$. State your answer to the nearest pence. **[7]**

(d) Suppose that we let the strike price K vary and keep the other parameters the same. What is the smallest value of K for which the call would has value zero? Explain your answer. **[6]**

End of Paper.

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