

Main Examination period 2022 – January – Semester A

MTH6154 / MTH6154P: Financial Mathematics I

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final.**

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **your must submit your exam within the first 3 hours of the assessment period.**

Examiners: Dr. D. Stark, Dr. J. Griffin

Question 1 [13 marks].

- (a) If you deposit £350 in a bank account with nominal interest rate 2.1% compounded semi-annually and make no withdrawals, how much money will be in the account three and a half years later? State your answer to the nearest pence. [3]
- (b) What is the effective rate of a bank account which is continuously compounded at rate 4.5%? State your answer as a percentage to three significant figures. [3]
- (c) Let $r(t)$, $t > 0$, be an instantaneous interest rate which is a non-decreasing and differentiable. Show that the associated yield curve $\bar{r}(t)$ is also non-decreasing. [7]

Question 2 [6 marks].

Explain briefly **in your own words** what is meant by **inflation adjusted interest rate** and the effect of using the inflation adjusted interest rate instead of the current interest rate when calculating the present value of a cash flow for which all payments are positive. [6]

Question 3 [11 marks]. Consider a 3 year bond with face value £150,000 with annual coupons paid at rate 5.1% and redeemable at face value.

- (a) Determine what is the payment stream of the bond. [3]
- (b) Determine the effective duration of the bond's payments if interest is compounded annually at rate 3%. State your answer to three significant figures. [4]
- (c) Find the duration of the bond's payments if interest is compounded continuously at rate 3%. State your answer to three significant figures. [4]

Question 4 [12 marks]. Suppose you decide to model interest rates by using the Varying Interest Rate model where R_i is the effective interest rate for a deposit between time i and time $i + 1$ and the R_i are an i.i.d. sequence $\{R_i\}_{i=0,1,2,\dots}$ of random variables with common distribution such that $1 + R_i$ is Lognormal(0.03, 0.0004) distributed. Suppose an amount £500 is deposited at time 0.

- (a) Determine the probability that the money accumulated at time $i = 10$ is greater than £750. State your answer to three significant figures. [5]
- (b) Determine the smallest time i such that the probability is larger than $1/2$ that the money accumulated is greater than £750. [7]

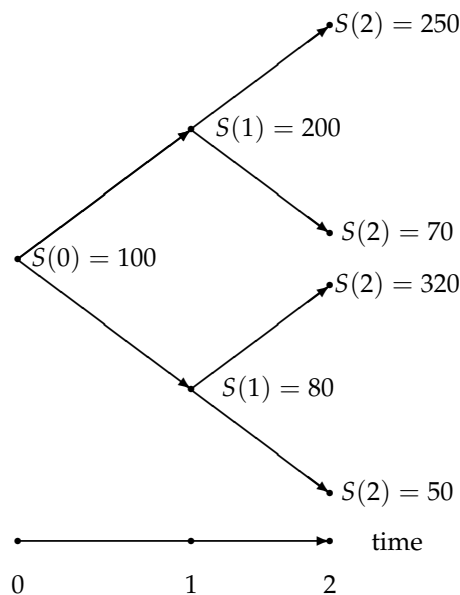
Question 5 [15 marks]. Assume that ABC Bank quotes spot rate rates $s_2 = 2\%$ and $s_7 = 2.2\%$.

- (a) Find the forward rate $f_{2,7}$. State your answer as a percentage to three significant figures. [4]
- (b) Suppose that an investor agrees to a Forward Rate Agreement with ABC Bank in which she deposits £300 in two years and to withdraw after a further five years. What should be amount she withdraws. State your answer to the nearest pence. [4]
- (c) Supposing that ABC Bank offers a higher effective rate to deposit money between 2 and 7 years from now than the rate you found in (a), explain in detail how you could construct an arbitrage. You may assume that zero-coupon bonds can be bought and (short) sold in any amount. [7]

Question 6 [11 marks]. Let x denote the number that is the last **two** digits of your student ID. You model a stock price $S(t)$ using a stochastic process, with t measured in years. Your model implies that the risk-neutral distribution for the stock price $S(4)$ at $t = 4$ is uniformly distributed over $[190 - x, 200 + x]$. Assume that interest is compounded annually at rate $r = 2\%$.

- (a) Write down the pay-off (as a function of $S(4)$) of a European put option with strike $K = 195$ and expiry $T = 4$, and sketch the graph of this function on the domain $[0, 210]$. [4]
- (b) Calculate the no-arbitrage price P of the put option in part (a). [7]

Question 7 [32 marks]. A share price in a two-period binomial model evolves according to this diagram:



Assume that interest is compounded continuously at rate 3%.

- (a) Verify that the no-arbitrage assumption is valid in this model. [6]
- (b) Find the risk-neutral probabilities of up and down movements in the share price at the nodes at time 0 and time 1. State your answers to four significant figures. [6]
- (c) Find the no-arbitrage price of a European call option on the share with strike $K = 290$ and expiry date $T = 2$. State your answer to three significant figures. [10]
- (d) Use the put-call parity formula and your answer to (c) to find the no-arbitrage price of a European put option on the share with strike $K = 290$ and expiry date $T = 2$. State your answer to three significant figures. [3]

Suppose, in addition, that the real world probability that the share goes up in the first step is 0.55.

- (e) (i) Explain whether or not this additional information changes your answer to part (c)? [3]
- (ii) Find the expected value of the price of the call option in part (c) at time 1. State your answer to three significant figures. [4]

End of Paper.