Queen Mary
University of London

Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH5125: Actuarial Mathematics II

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work, you should upload photographs or scans of your work using the upload tool on the QMplus page for the module. You should also email a copy of your work to maths@qmul.ac.uk with your student number and the module code in the subject line.

One of the photographs you upload should feature the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems.

IFoA exemptions
This module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, you must submit within the first **3 hours** of the assessment period. You may then submit a second version later in the assessment period if you wish, which will count only towards your degree.

Examiners: C. Sutton, A. Baule

Question 1 [19 marks]. A whole life assurance for a life age 56 has a sum assured of £50,000 payable at the end of year of death and is secured by level annual premiums paid at the beginning of each policy year. The life assurance company assumes initial expenses of £500, renewal expenses of 5% of each premium and termination expenses of £50.

(a) Write down the equation of value for this policy. [**4**]

(b) Calculate the annual premium if $A_{56} = 0.358$ and the interest rate is 3% p.a. [**5**]

(c) Write down the equation for the gross future loss variable for this policy. [**2**]

(d) Suppose mortality follows the life table below and interest is 3% per annum. What is the smallest annual premium that leads to the probability of the life assurance company making a loss being less than 10%? [**8**]

Age x	56	57	58	59	60	61	62	63	64	65
l_x	8456	8425	8395	8316	8265	8139	7996	7805	7626	7436

Question 2 [19 marks]. A life assurance company offers a policy to two lives both age 60 which pays a sum assured of £150,000 at the end of the policy year following the second death for which annual premiums are payable whilst at least one of the policyholders is alive. The company assumes initial expenses of £850 plus renewal expenses of 6% of each premium, and uses an interest rate of 3% p.a.

(a) Calculate the gross premium using the equivalence principle and the whole life assurance values from the table below. [**10**]

x	A_x	$A_{x:x}$
60	0.608	0.791
65	0.658	0.822
70	0.729	0.874

(b) Calculate the prospective gross premium reserve after 5 years on the same basis as (a) above assuming both lives are still alive at the time. [**6**]

(c) How would the reserve calculation need to change if one of the lives had died in the first 5 years of the policy? [**3**]

Question 3 [13 marks]. Whenever two people borrow money to pay for a property by taking a mortgage with Gamma Bank Ltd they are advised to purchase a joint life term assurance policy from Theta Insurance Co with a term equal to that of the mortgage and a sum assured equal to the original loan amount.

Two lives age x and y buy a property for £250,000 and borrow 95% of the purchase price from Gamma Bank. Theta Insurance pay Gamma Bank sales commission of 20% of the first year's premium and assume that other expenses are 4% of each annual premium.

(a) Write down the Equation of Value for the life assurance policy. [**4**]

(b) Write an expression for the Prospective Gross Premium Reserve after t years. [**4**]

(c) Under what conditions will the Prospective and Retrospective Gross Premium Reserves be equal? [**3**]

(d) Explain why the Prospective Gross Premium Reserve will increase if actuaries at Theta Insurance lower their assumption for long term interest rates part way through the term of this policy. [**2**]

Question 4 [18 marks]. Arthur retires on his 55th birthday when, as part of the benefits of the pension scheme he is a member of, he receives a lump sum of £25,000. He would prefer to convert this into an annuity payment. A life assurance company charges 2% initial expenses on all of their annuity products and they assume an interest rate of 4% per annum.
The mortality table they use to value these contracts gives $A_{50} = 0.385$, $A_{55} = 0.449$, and $A_{55:50} = 0.503$.

(a) Calculate the amount of single life annuity payable annually in advance that Arthur would receive on this basis. [**5**]

(b) How much would the annual annuity reduce by if Arthur and Betty (who is exactly 5 years younger) used the lump sum to purchase a last survivor annuity instead? [**4**]

(c) An actuary suggests that Arthur and Betty consider combining two annuities which pay annually in arrears: a life annuity for Arthur and a reversionary annuity with Betty as annuitant receiving 50% of the annual payments under the first annuity. Calculate the annuity amounts that could be purchased with the retirement lump sum under this arrangement. [**9**]

Question 5 [11 marks]. Harry has a model railway where a miniature train goes around a circular track powered by a low voltage electric current. Once Harry turns the train on it will continue to go around the track loop until either:

- the train is derailed with the wheels coming off the track, or
- there is an interruption to the electricity supply to the track

If either of these occur the train will stop immediately.

(a) Draw a diagram for a 3-state model clearly labelling transition intensities for the two ways in which the train may stop. [**1**]

(b) If the derailment transition intensity is 0.03 and the electric supply failure intensity is 0.05, find the probability that the train successfully completes 10 loops of the circular track. [**4**]

(c) Harry seeks to improve the electricity supply by introducing two power sources such that the train stops once both sources fail. One linked to a lithium battery has a failure transition intensity of 0.015 and the other linked to a solar panel has a failure transition intensity of 0.025. If we ignore the risk of derailment, what is the expected number of completed laps before the train stops due to electricity supply failure? [**6**]

Question 6 [10 marks]. An insurance company offers policies which include both life assurance and critical illness cover. It calculates premiums using a multiple decrement table. The forces of decrement due to death μ_x^d and critical illness μ_x^c are given in the table below.

Age x	μ_x^d	μ_x^c
50	0.0026	0.0005
51	0.0028	0.0006
52	0.0032	0.0008

(a) Construct a multiple decrement table for ages 50, 51, 52 and 53 with a radix equal to your 9 digit QMUL Student ID number. So, if for example your QMUL Student ID is 140470173 you should set $(\text{al})_{50}$ to be 140,470,173. [**8**]

(b) Comment on the assumptions that underlie the multiple decrement table and how realistic these are. [**2**]

Question 7 [10 marks]. A life assurance company issues a two-year unit-linked endowment assurance to a life age 65 exact. If the policyholder dies during the term of the policy the death benefit, payable at the end of the year of death, is the bid-value of units subject to a minimum of £20,000. On survival to the end of the two-year term the bid-value of units is paid. Premiums of £10,000 are paid at the beginning of each policy year and 99% are allocated to units. Units have a bid-offer spread of 1% and a management charge of 0.5% is deducted at the end of the policy year before the payment of any benefits. The life assurance company assumes that per-policy expenses are £150 in the first year and £75 in the second year.

When carrying out a profit test, the life assurance company assumes that investment returns in the unit fund are 8% p.a. and annual interest on cash balances is 1.5%. Mortality assumptions are given in the table below.

Age x	65	66
q_x	0.0034	0.0038

(a) Produce projected revenue accounts for each policy year. [**8**]

(b) Calculate the profit vector for this policy. [**2**]

End of Paper.