MTH5125: Actuarial Mathematics II

Duration: 2 hours

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> **You should attempt ALL questions. Marks available are shown next to the questions.**

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Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiners: J.Webber, L.Zhao

Question 1. [17 marks] A life insurance company calculates premiums for a 25-year with profit endowment assurance on the basis below. Benefits (including any bonus) are paid immediately on death. The sum insured on every policy is £10,000 and premiums are payable annually in advance. Compound reversionary bonus is added at the end of each policy year.

(a) Show that the premium paid by a 35 year old is £315.32 per annum. [7]

(b) Determine the sum insured plus reversionary bonus after 11 years if reversionary bonus rates are 3% for the first three years, 2% for the next three years and 1% p.a. thereafter. [3]

(c) Show that the gross premium reserve for the policy in part (a) after 11 years is £4,217. The reserve is determined assuming an interest rate of 4% p.a. and using the same assumptions for expenses and mortality as the premium basis; no allowance is made for any future declaration of bonus. [7]

Basis:

Mortality	AM92 Select
Interest	6% p.a.
Initial Expenses	60% of the first year's premium; incurred at the start of the contract
Renewal Expenses	5% of the annual premium from year 2 onwards; incurred at the start of each year
Bonus loading	1.923 p.a.% compound bonus

Question 2. [12 marks]

(a) Let R be a random variable representing the present value of a payment of 1 at the end of year of death of a life aged x. Show from first principles that the variance of R can be written as:
$$Var(R) = {}^2A_x - (A_x)^2$$
where ${}^2A_x$ is calculated at $i' = (1+i)^2 - 1$ and where A_x is the expected value of R. [4]

(b) Let T be a random variable representing the present value of an annuity of 1 paid annually in advance until the death of a life aged x. Show from first principles that the variance of T can be written as:
$$Var(T) = \frac{1}{d^2}\left[{}^2A_x - (A_x)^2\right].$$

[4]

(c) A whole of life policy with a sum insured of S payable at the end of year of death of a life aged x has premiums of P payable annually in advance for the lifetime of the life assured. Show that the variance of the present value of future losses (ignoring expenses) at the start of the contract can be written as:
$$Var(L) = \left[S + \frac{P}{d}\right]^2 \left({}^2A_x - (A_x)^2\right).$$

[4]

Question 3. [18 marks]

Employees in a high risk job are subject to three types of decrement from work: mortality (D), invalidity (I) and withdrawal from employment (W). There is no return to work after invalidity or withdrawal. All employees are required to retire at age 40.

The independent forces of decrement are independent of age and are given in the table below.

Age	μ_x^D	μ_x^I	μ_x^W
$x < 40$	0.0050	0.020	0.10

(a) Draw and label a transition state diagram for the situation above.   **[4]**

(b) Copy and complete the multiple decrement table below by calculating $(ad)_{30}^D$, $(ad)_{30}^I$, $(ad)_{30}^W$ and $(al)_{31}$.

x	$(al)_x$	$(ad)_x^D$	$(ad)_x^I$	$(ad)_x^W$
30	10,000			
31		n/a	n/a	n/a

                                                                                      [6]

(c) Show that the probability that a new employee aged x will be working in t years time is $e^{-0.125t}$ for $x + t < 40$.   **[4]**

(d) Employees receive a lump sum benefit immediately on invalidity of £100,000. Calculate the expected present value of this benefit for a new employee aged 25 exact assuming a force of interest of 0.06 p.a..   **[4]**

Question 4. [18 marks]

(a) Explain the meaning of the term $\ddot{a}_{xy}^{(6)}$.   **[4]**

(b) Estimate $\ddot{a}_{65:60}^{(6)}$ for two lives A (age 65) and B (age 60) on the basis below.   **[5]**

(c) Explain the meaning of the term $_{10}q_{65:60}$.   **[4]**

(d) Determine $_{10}q_{65:60}$ on the basis below.   **[5]**

Basis:
  Mortality $\mu_x = 0.03$ for A
                      $\mu_x = 0.04$ for B
  Interest $\delta = 0.06$ p.a.
  Expenses   Ignore

Turn Over

Question 5. [18 marks] A life insurance company issues a 3-year unit linked endowment assurance policy to a person aged 55. The details are as follows:

- Premiums of £2,000 payable annually in advance.

- 85% of the premium is allocated to units in the first policy year and 100% thereafter.

- Units have a bid/offer spread of 5%.

- A policy fee of £25 is deducted from the unit fund at the start of each year.

- An annual management charge of 1.5% of the bid value of units is deducted at the end of each year, before any benefit payments.

- The benefit on death is guaranteed to be at least 5 times the annual premium.

- The benefit on maturity or surrender is 100% of the bid value of units.

- Death and surrender benefits are payable at the end of year only.

The profit vector for the first three policy years is $(-£34.32, £96.84, £137.28)$.

    (a) Define the terms **profit vector** and **profit signature**.   **[4]**

    (b) Assuming the mortality rates and surrender rates given below are independent probabilities of decrement, and assuming a constant force of decrement for each decrement in a policy year, determine the following dependent probabilities: $(ap)_{55}$, $_2(ap)_{55}$, and $(aq)_{55}^{(D)}$. These probabilities will be required for parts (c) and (d) below.   **[3]**

    (c) Calculate the profit vector element for the first policy year using the profit testing assumptions below. Show all of your working with explanation. (The answer should be $-£34.32$ as shown in the profit vector above.)   **[8]**

    (d) Determine the present value of profits for the policy using the profit vector above.   **[3]**

Profit Test Assumptions

Rate of growth of assets in unit fund	7% p.a.
Rate of interest on non-unit fund cashflows	2% p.a.
Mortality	AM92 Ultimate
Surrender	Probability of surrender is 15% in year 1 and 5% p.a. thereafter
Initial Commission	10% of the first premium
Renewal Commission	1% of all premiums after the first
Initial Expenses	£200
Renewal Expenses	£35 at the start of each policy year
Rate of expense inflation	2.5% p.a. from year 1
Risk discount rate	8% p.a.

Question 6. [17 marks]

(a) Define what is meant by the following forms of selection and give an example of each.

- Adverse selection [**4**]

- Spurious selection [**4**]

(b) The country of Outland has a region called Stayin. A mortality survey for the years 2015-17 is summarised in the table below.

Age Group	Stayin Exposed to Risk	Stayin Deaths	Outland Exposed to Risk	Outland Deaths
20-39	500,000	405	1,000,000	900
40-59	400,000	490	1,500,000	2,100
60-79	100,000	480	2,000,000	9,500
Total	1,000,000	1,375	4,500,000	12,500

(i) Calculate the indirectly standardised mortality rates for Stayin and Outland, using the Outland mortality rates as the standard rates. [**6**]

(ii) Explain the differences between the standardised and indirectly standardised mortality rates for an area and discuss which might be the most appropriate approach for comparing mortality rates between regions. [**3**]

End of Paper.