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Main Examination period 2017

MTH6116 / MTH6116P: Design of Experiments
Duration: 2 hours

Student number

Desk number

Name and type of calculator used _____

Examiners: H. Maruri-Aguilar, R. Johnson

Turn Over

This page is for marking purposes only.
Do not write on it.

Question	Mark	Comments
1		
2		
3		
4		
Total		

Question 1. **[22 marks]** A chemical was used to treat fungi pest in a certain type of tree. The study involved two formulations of the chemical and four trees. After spreading each chemical into a tree, the amount of fungi pest was measured in two branches per tree. The following table shows the design plan, after suitable randomization.

Branch	Tree	Chemical
1	1	B
2	1	B
3	2	A
4	2	A
5	3	A
6	3	A
7	4	B
8	4	B

In what follows, work over the set of plots $\Omega = \{1,2,3,4,5,6,7,8\}$ and consider the usual factors U and E≡Branch.

(a) Build equivalence classes for plot factor Tree, and for treatment factor Chemical.   **[8]**

(b) Build relations between treatment and plot factors, construct a combined Hasse diagram and compute the degrees of freedom.   [**8**]

(c) Using your results, build an anova table in which you specify strata, sources of variation and write the degrees of freedom.   [**4**]

(d) Briefly explain why factors `Chemical` and `Tree` are orthogonal.   **[2]**

Question 2. [30 marks]
An industrial experiment was performed to study the effects of two levels of Carbon (20 and 50 grams) and two levels of Sulphur (10 and 15 grams) and their interaction on the yield of a vulcanization process. A randomized block design was used; the design and data are shown below:

		Block 1				Block 2		
Carbon	50	50	20	20	20	50	50	20
Sulphur	10	15	10	15	10	10	15	15
Yield	86	87	60	57	55	89	86	52

(a) Briefly explain how you would randomize a design like the one shown. **[4]**

(b) Draw two Hasse diagrams, one for the factors on the observational units and one for the factors on the treatments. Add degrees of freedom to your diagrams. **[8]**

(c) For the data in the table, the grand total of the yields is equal to 572 and the sum of the squared response values is equal to $\sum_{\omega} y_{\omega}^2 = 42860$. Moreover, the totals for blocks, levels of carbon, levels of sulphur and the treatments are:

Block		Carbon		Sulphur		Treatments			
1	2	20	50	10	15	(20,10)	(20,15)	(50,10)	(50,15)
290	282	224	348	290	282	115	109	175	173

Compute the analysis of variance table (anova) for the experiment assuming that blocks have fixed effects. There is additional space for calculations on the following page. **[8]**
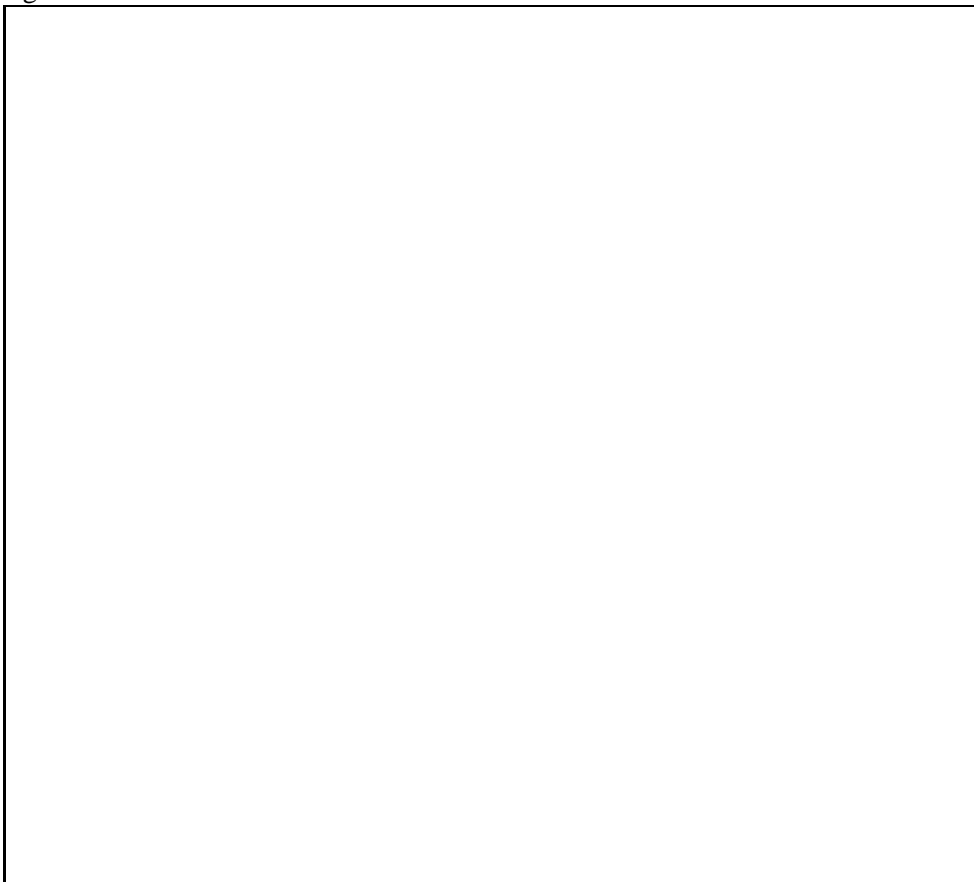
(Additional space for anova calculations)

(d) Test whether the main effects of Carbon and Sulphur and their interaction are significant at the 5% level.   [**6**]

(e) To analyse the data with GenStat, what factors must be declared?   [**2**]

(f) How would you specify the structures GenStat needs to know for carrying out the analysis of variance? Fill the GenStat dialogue boxes below.   [**2**]

Treatment Structure:

Block Structure:

Question 3. [24 marks]

A process engineer is interested in constructing plastic pieces with high resistance to impact. He performed an experiment in which four different formulas of plastic A, B, C and D were hardened. The oven has four shelves, and each shelf has four positions thus he considered a row-column design in which rows correspond to shelves and columns correspond to positions. The experimental arrangement is shown below, together with measurements of resistance to impact.

<div align="center">

Position

	1	2	3	4
1	D	C	A	B
2	B	A	C	D
3	C	B	D	A
4	A	D	B	C

Shelf (rows 1–4)

Position

	1	2	3	4
1	277	267	259	265
2	263	263	277	283
3	273	263	277	251
4	269	281	263	269

Shelf (rows 1–4)

</div>

(a) Explain what is meant by a row-column design. **[4]**

(b) Define what a **Latin square** is. **[4]**

(c) Complete the anova table for the experiment. To this end, use the space below for calculations and write your answer in the boxes provided. Additional space for calculations is provided on the following page. [**13**]
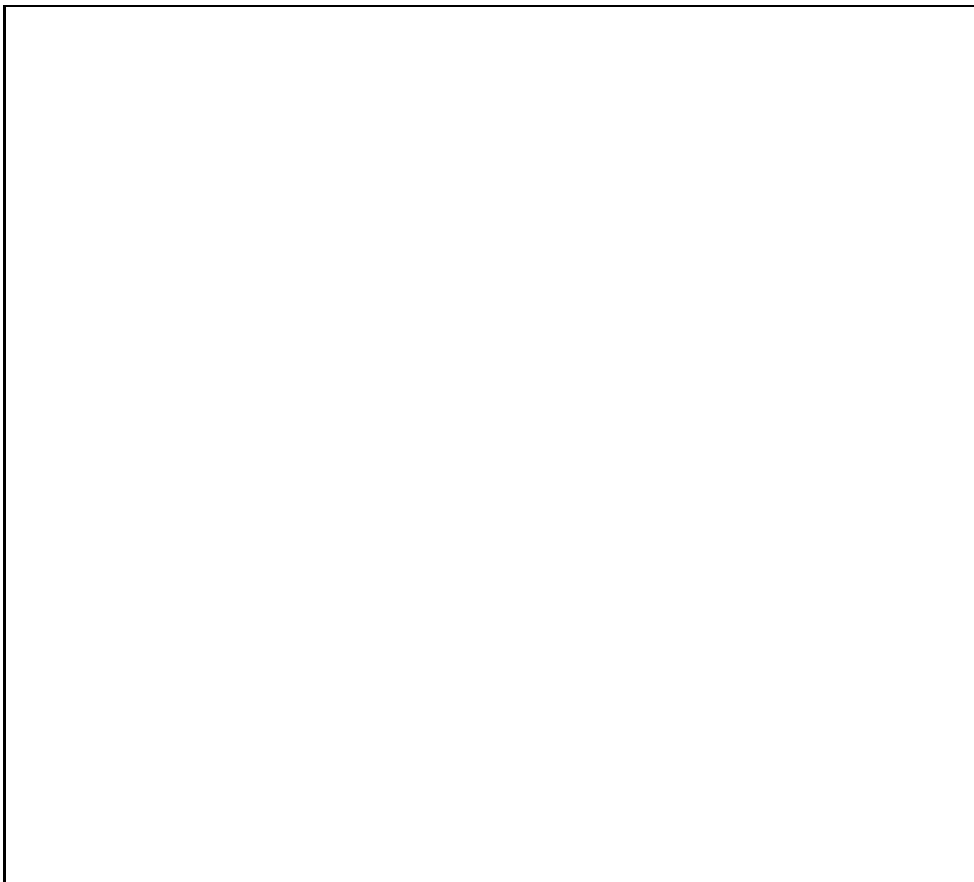
Stratum	Source	SS	d.f.	MS	VR
Mean	Mean	1155625	☐	☐	....
Shelf	Shelf	☐	☐	☐	....
Position	Position	☐	3	☐	....
Plot	Formula	☐	☐	☐	☐
	Residual	150	6	☐	....
	Total	1156760	☐		

(Additional space for anova calculations)

(d) Using a significance level of 5%, perform the hypothesis test for the effect of factor
formula.   [**3**]

Question 4. **[24 marks]** An experiment on methods of controlling the disease sclerotina in oilseed rape compared four new chemical solutions, coded A, B, C and D with both 'no treatment' and the current (expensive) standard solution X. Each of the new solutions could be applied early or late; the standard solution X was applied at both times. Thus there were two control treatments, and the treatments have the structure shown in the following table, where the numbers in the table identify treatments.

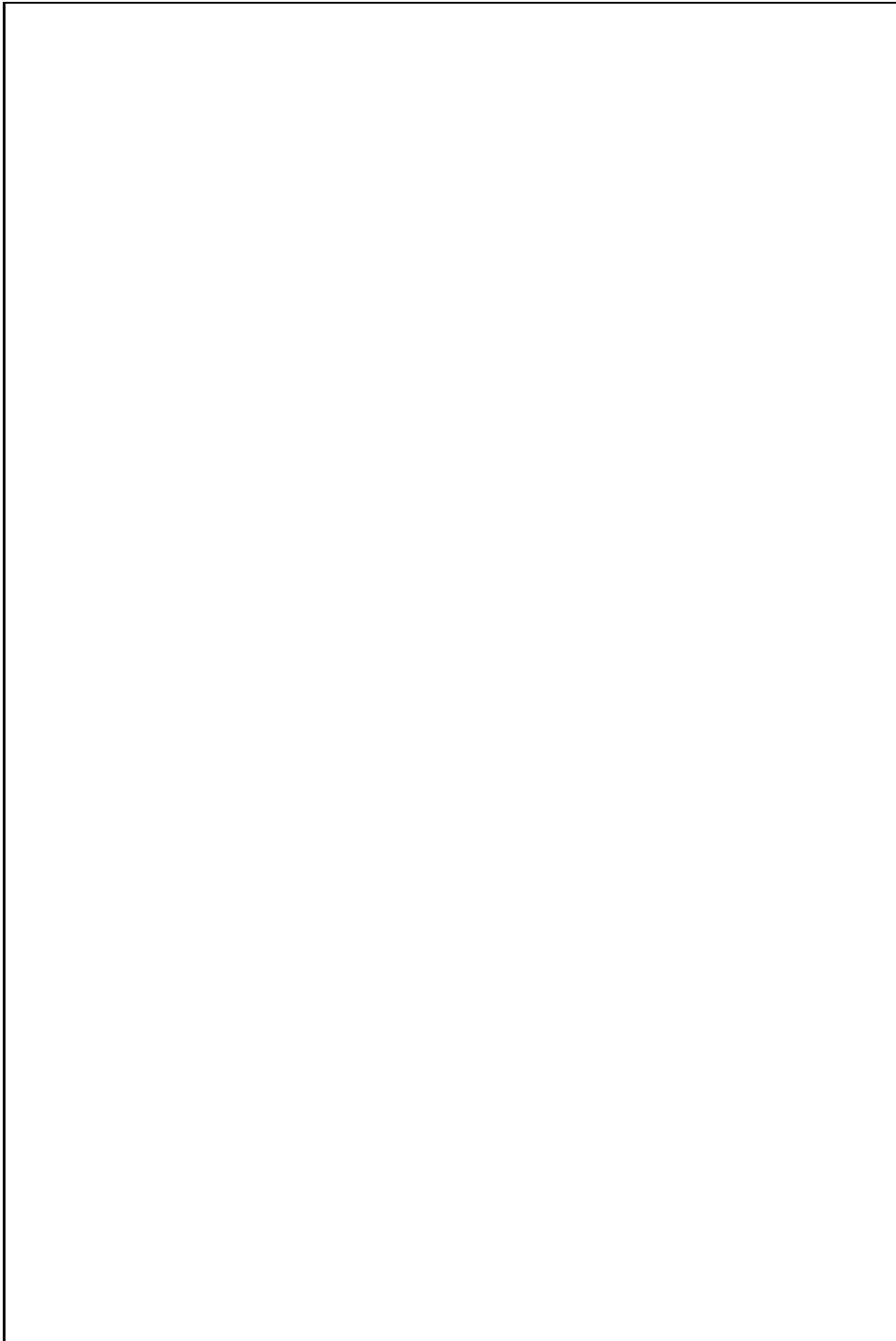|		Time			
		early	late	both	n/a
Solution	none				10
	A	1	5		
	B	2	6		
	C	3	7		
	D	4	8		
	X			9	

In what follows, work over the set of treatments $\mathscr{T} = \{1,2,3,4,5,6,7,8,9,10\}$ with the usual factors U and Treatment≡E.

  (a)   Build equivalence classes for treatment factors Solution and Time.   **[8]**
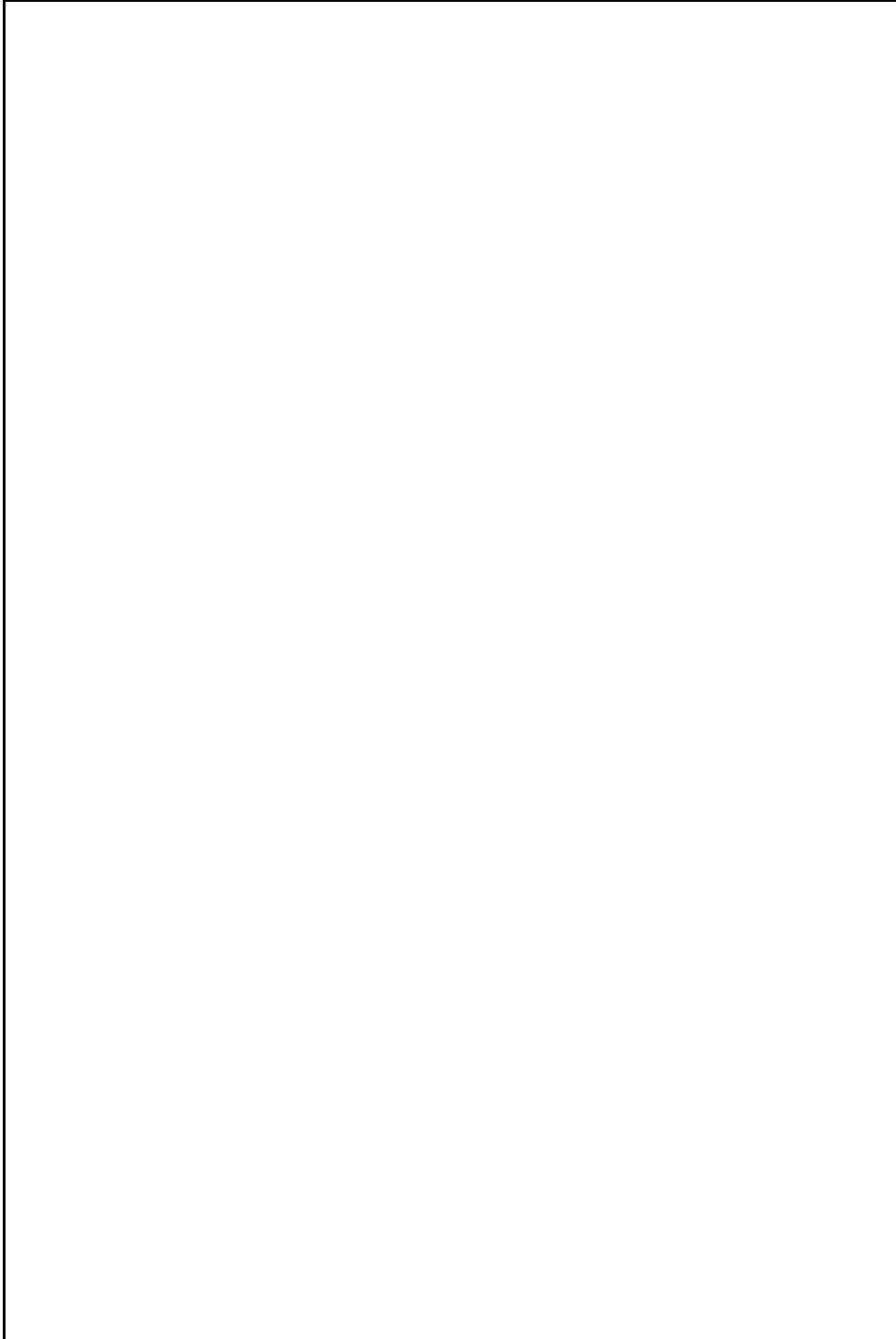
(b) Also build equivalence classes for the supremum $\texttt{Solution} \lor \texttt{Time}$ and for the infimum $\texttt{Solution} \land \texttt{Time}$.   **[8]**
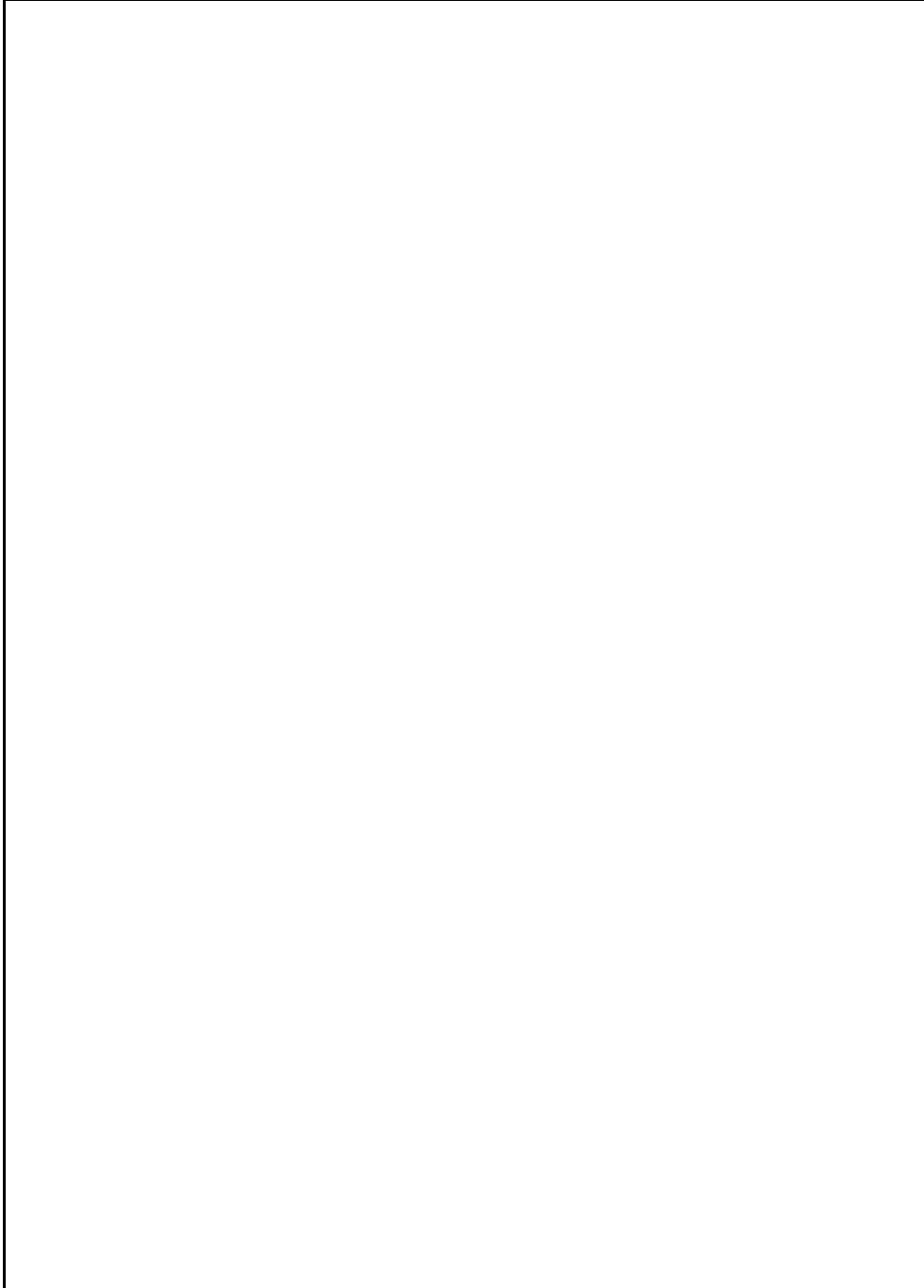
(c) The factor `Applied` was defined to distinguish whether some solution (A, B, C, D or X) had been applied or not. Build equivalence classes for this factor and show that $\text{Solution} \vee \text{Time} \prec \text{Applied}$.   **[4]**

(d) Build the Hasse diagram using all factors so far considered over $\mathscr{T}$. Add the degrees of freedom to your diagram.　　　　　　　　　　　　　　　　　　　　　　**[4]**

End of Paper.