B. Sc. Examination by course unit 2014

MTH6100   Actuarial Mathematics

Duration: 2 hours

Date and time: 2nd May 2014, 14:30–16:30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. The unauthorized use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

A sheet of formulae and certain life tables are provided for this examination, attached to the examination paper.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): D. Stark, R. Harris

Question 1

(a) Deborah Dough invests £15,000 in a savings account for which the effective interest rate is 1.2% per annum. Find, to the nearest penny, the accumulation after three years. [3]

(b) Adam West invests £12,000 in a savings account for which the nominal rate of interest is constant at 3.5% per annum compounded semi-annually. Find, to the nearest penny, the accumulation after six years and six months. [3]

(c) (i) State the relationship between the effective rate of interest i and the nominal rate of rate of discount $d^{(p)}$. [2]

   (ii) If the effective rate of interest per annum is 6%, then how much interest (to the nearest penny) must be paid in advance to borrow £900 for one month when interest is compounded monthly? [3]

(d) Johnny Appleseed is paying back a loan for £12,000 taken out exactly five years ago and being paid annually in arrears for 12 years. The fifth payment has just been made. The annual interest rate charged on the loan is 7%.

   (i) Find the yearly payment (to the nearest penny). [5]

   (ii) Calculate the amount of the loan which is still outstanding (to the nearest penny). [4]

(e) State what is meant by the actuarial symbol $(I\bar{a})_{\overline{n}|}$. [3]

Question 2

(a) Assuming the mortality given by table A1967-1970 select values, calculate the probability $p_{[42]}$ to 4 decimal places. [4]

(b) Suppose that a newborn has lifetime distributed according to random variable X with survival function $s(x) = P(X > x)$.

   (i) Show that the curtate further lifetime $K(x)$ satisfies

$$P(K(x) = k) = \frac{s(x + k) - s(x + k + 1)}{s(x)}.$$

[5]

   (ii) Show that the expected curtate further lifetime e_x satisfies

$$e_x = \frac{1}{l_x} \sum_{k=1}^{\infty} l_{x+k}$$

[6]

(c) Suppose that the values $p_0 = 0.4$, $l_0 = 7500$ are known for a particular life table. Find q_0, l_1 and d_0. [3]

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Question 3 It is observed that for the race of Sneeches, the force of mortality at age x years is given by

$$\mu(x) = \frac{1}{200 - 2x} \quad \text{for } 0 \le x \le 100.$$

(a) Find $s(x)$ for $x \ge 0$. [4]

(b) Find the probability that a Sneech of age 50 dies before age 60 (stated to four decimal places). [3]

(c) Find the probability that a Sneech of age 40 lives to be age 50 but dies before age 55 (stated to four decimal places). [4]

(d) Find the probability density function $f_X(x)$ for $x \ge 0$. [3]

(e) Given $0 \le x < 100$, find the probability density function $f_{T(x)}(t)$ for $t \ge 0$. [3]

Question 4 Give your answers to the following questions to the nearest penny. Assume an effective annual interest rate of 4% and the mortality given by table A1967–70 select values. Answer all questions to the nearest penny.

(a) At age 45, Jane Smith takes out a whole-life assurance, with a death benefit of £120,000 payable at the end of the year of her death.

   (i) She pays for this with an annual premium in advance while she is alive. How much is her payment? [6]

   (ii) Jane lives to be 60 and at that time surrenders her policy for a lump sum. How much does she receive? [7]

(b) At age 52, Billie Brown purchases in one payment a twelve year endowment policy paying a death benefit of £40,000 if she dies within twelve years and £40,000 if she survives twelve years. What is the cost of her policy? [6]

Question 5

(a) Consider an investment fund with the following history:

- On 1 January 2013, the value of the assets was 1.1 hundred million dollars.
- On 31 March 2013, the value of the assets was 1.3 hundred million dollars.
- On 1 April 2013, there was a net cash inflow to the fund of 0.8 hundred million dollars.
- On 31 December 2013, the value of the assets was 2.0 hundred million dollars.

  (i) Find the money weighted rate of return of this fund for 2013. Use linear interpolation and the fact that the money weighted rate of return lies between 5% and 6%. State your answer as a percentage to two decimal places. [6]

  (ii) Find the time weighted rate of return of this fund for 2013. State your answer as a percentage to two decimal places. [4]

  (iii) Briefly explain why the money weighted rate of return is significantly lower than the time weighted rate of return. [2]

(b) (i) State what is meant by the Principle of Consistency. [2]

  (ii) Ganesh Patel puts £5000 in the bank at time $t = 0$ and does not take any money out until time $t = 1$, where t is measured in years. The force of interest is
  $$\delta(t) = \frac{1}{25(1 + t)}.$$
  How much money does Ganesh take out at time $t = 1$? State your answer to the nearest penny. [4]

(c) Under the assumptions that force of interest is constant δ and that $T(x)$ is Exponential(μ) distributed, show that
$$\bar{A}_x = \frac{\mu}{\mu + \delta}.$$

[5]

End of Paper—An appendix of 8 pages follows (formulae and tables)