MTH5123 - DIFFERENTIAL EQUATIONS - 2021/22

YOU CAN PREVIEW THIS QUIZ, BUT IF THIS WERE A REAL ATTEMPT, YOU WOULD BE BLOCKED BECAUSE:

This quiz is not currently available

QUESTION 1

Not yet answered Marked out of 20.0

For each multiple choice question select only one of the options:

a) Which of the following ordinary differential equations (ODEs) has t as the independent variable and can be solved by the separation of variables method? (4 marks)

○I ○II ○III ○IV

where I: $\frac{dy}{dx} = e^y \sin(y)$, II: $\dot{y} = t + e^{t+y}$, III: $\dot{y} = t^2 \cos(y+5)$, IV: $\dot{y} = e^{t^2+y}$

b) Which of the following ODEs is not separable but can be reduced to be separable? (4 marks)

○I ○II ○III

where I: $y' = 3\ln(y) - 3\ln(x) + 10\frac{y}{x}$, II: $y' = \tanh((y/x)^2) + 2e^{x/y}$, III: $\dot{y} = (t+3y)t$.

c) Which of the following statements is correct for solving the ODE, $\dot{x} = 3\cos(5x + 7t - 2)$? (4 marks)

○The ODE can be reduced to be separable;

○This is an inhomogeneous linear ODE that can be solved by the variation of parameter method;

○The method to solve exact ODEs can be applied;

d) The solution to the initial value problem $y' = \frac{3}{\sin(y)}$, $y(0) = \pi/2$ is, (8 marks)

○I ○II ○III ○IV

where

I: $y(x) = \arcsin(C + 3x)$, where C is an arbitrary constant,

II: $y(x) = \arcsin(C - 3x)$, where C is an arbitrary constant,

III: $y(x) = \arccos(3x)$

IV: $y(x) = \arccos(-3x)$

Find the right match for the following ODEs in the dropdown menu

$y'' = 5x + 3y + 2$	Choose...
$y = y'' \sin(x) + e^x + 2$	Choose...
$3xy' = y - 2x^2 y''$	Choose...
$\dot{y} = e^t + t^2 y$	Choose...
$y' = \sin(x/y)$	Choose...

QUESTION 3

Not yet answered Marked out of 20.0

a) Consider the initial value problem (IVP) $y' = yx/(x^2 - 1), \quad y(0) = 1$. Does this IVP satisfy the hypotheses of the Picard-Lindelöf theorem? (5 marks)

- ○Yes because the function $f(x, y) = yx/(x^2 - 1)$ is continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates$(x_0, y_0) = (0, 1)$.
- ○No because the function $f(x, y) = yx/(x^2 - 1)$ is not continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates$(x_0, y_0) = (0, 1)$.
- ○Yes because the function $f(x, y) = yx/(x^2 - 1)$ and its partial derivative $\partial f(x, y)/\partial y$ are continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates $(x_0, y_0) = (0, 1)$.
- ○No because neither the function $f(x, y) = yx/(x^2 - 1)$ nor its partial derivative $\partial f(x, y)/\partial y$ are continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates $(x_0, y_0) = (0, 1)$.

b) What is the solution to the initial value problem in point (a) valid sufficiently close to the initial condition? (5 marks)

- ○I
- ○II
- ○III

where $\mathrm{I}: y(x) = 2\sqrt{|x^2 - 1|}, \mathrm{II}: y(x) = \sqrt{x^2 - 1}, \mathrm{III}: y(x) = \sqrt{1 - x^2}$.

c) Consider the initial value problem (IVP) $y' = yx/(x^2 - 1), \quad y(1) = 0$. Does this IVP satisfy the hypotheses of the Picard-Lindelöf theorem? (5 marks)

- ○Yes because the function $f(x, y) = yx/(x^2 - 1)$ is continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates$(x_0, y_0) = (1, 0)$.
- ○No because the function $f(x, y) = yx/(x^2 - 1)$ is not continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates $(x_0, y_0) = (1, 0)$.
- ○Yes because the function $f(x, y) = yx/(x^2 - 1)$ and its partial derivative $\partial f(x, y)/\partial y$ are continuous in a sufficiently small rectangular region D centred in the point of the xy plane of coordinates $(x_0, y_0) = (1, 0)$.

d) How many solutions does the IVP in point (d) have? (5 marks)

- ○None
- ○1
- ○2
- ○Infinitely many

(a) Consider the boundary value problem (BVP) $5y'' + 45y = -\tan(7x)$, $y(0) = 0$, $y(\pi/7) = 1$. Does this BVP have a unique solution ? (5 marks)

- ○Yes
- ○No
- ○It is impossible to determine the ODE cannot be solved.

(b) For which real value of b the following BVP $5y'' = -45y + \cos(3b)$, $y(0) = 0$, $y(\pi/3) = -5$ has a unique solution ? (5 marks)

- ○$b = n\pi/2$ with n integer
- ○$b \neq n\pi/2$ with n integer
- ○Any value of b
- ○No value of b

QUESTION 5

Not yet answered Marked out of 10.0

(a) Consider a system of two ordinary differential equations: $\dot{y}_1 = \tan(\frac{1}{2}y_1 - y_2) - y_2^2$, $\dot{y}_2 = \sin(y_1) + \frac{1}{2}\sin(y_2)$. Linearise the system of ODE close to the $(y_1, y_2) = (0, 0)$ equilibrium. The phase portrait of the linearised system displays a: (5 marks)

- ○Stable node
- ○Unstable node
- ○Saddle
- ○Unstable focus with spiral out
- ○Centre
- ○Stable focus with spiral in

(b) For which real value of a the system of ODEs
$\dot{y}_1 = \sin(ay_2 + 2y_1)$, $\dot{y}_2 = -\tanh(y_1 + ay_2)$ when linearised displays a saddle at $(y_1, y_2) = (0, 0)$? (5 marks)

- ○$a < 2$
- ○$a > 0$
- ○$a < -2$
- ○$a < 0$

This question requires an handwritten answer which should be uploaded here in a the format of a single combined pdf.

a) Consider the ODE describing the motion of a pendulum in presence of friction. Let θ indicate the angle of the pendulum with respect to the vertical line and let t indicate the time.
The ODE describing the motion of the pendulum is given by

$$m\ell\ddot{\theta} = -mg\sin\theta - \gamma\dot{\theta},$$

with $\theta \in [-\pi/2, \pi/2]$. Here $m > 0$ indicates the mass of the pendulum, $\ell > 0$ indicates its length, $g > 0$ indicates the gravitational constant, and $\gamma \geq 0$ is a constant real parameter indicating the intensity of the friction.

- Identify the dependent and independent variable in this ODE. (1 mark)
- Is this a linear or non-linear ODE? (2 marks)
- What is the order of this ODE? (2 marks)

b) Consider again the ODE introduced in point (a) and describing the motion of the pendulum

$$m\ell\ddot{\theta} = -mg\sin\theta - \gamma\dot{\theta}.$$

with $\theta \in [-\pi/2, \pi/2]$. Put $m = 1$ and $\ell = 1$.

- Convert this ODE into a system of two first-order ODEs. (4 marks)
- Compute all equilibria of this system of ODEs.
  Linearise this system of ODE around each equilibrium.
  Find the eigenvalues of the linearised system around each equilibrium. (9 marks)
- Assume that $g > 0$ is constant but that $\gamma \geq 0$ can be tuned.
  For which values of γ are the phase portraits of the linearised systems fixed points?
  For which values of γ are the phase portraits of the linearised systems stable focuses?
  For which values of γ are the phase portraits of the linearised systems centres? (9 marks)
- Explain the meaning of your results. (3 marks)

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