Queen Mary
University of London

Main Examination period 2019

MTH5103: Complex Variables

Duration: 2 hours

Write your solutions in the space provided in this exam paper.

Student number: | | | | | | | | | |

Desk number: | | | |

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: M. Shamis, O. Jenkinson

This page is for marking purposes only:

DO NOT WRITE ON IT

Question	Mark	Subpart Breakdown
1		
2		
3		
4		
5		
TOTAL :		

Question 1. [20 marks]

(a) Compute the following number

$$\left|\frac{(2-i)(2+2i)^5}{(i+i^2+\cdots+i^{11})^{20}}\right| .$$

Justify all of your steps. [**10**]

Write your solution to Question #1(a) below

(b) Find all the solutions $z \in \mathbb{C}$ of the equation

$$z^{10} = 3 + 3i.$$

Please write the answer either in Cartesian or in polar form. [**10**]

Write your solution to Question #1(b) below

Additional space for Question 1

Question 2. [20 marks]

(a) State the Root (Cauchy) Test for complex series. [**7**]

Write your solution to Question #2(a) below

(b) Using the Root Test, or otherwise, determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} i^n z^n.$$

Justify all of your steps. [**10**]

Write your solution to Question #2(b) below

(c) Does the series in part (b) converge for any z with $|z| = 1$? Please justify your answer using an appropriate test for convergence or divergence. [**3**]

Write your solution to Question #2(c) below

Additional space for Question 2

Question 3. [20 marks]

(a) Determine the residue of the function

$$f(z) = \frac{1 - \cos z}{(z - 2\pi)^3}$$

at the point $z = 2\pi$. [**10**]

Write your solution to Question #3(a) below

(b) Find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n(z - z_0)^n + \sum_{n=1}^{\infty} b_n(z - z_0)^{-n}$$

for the function

$$f(z) = \frac{1 - \cos z}{(z - 2\pi)^3}$$

from (a) on the set $\{z \in \mathbb{C} : \; |z - 2\pi| > 0\}$ (for $z_0 = 2\pi$). **[10]**

Write your solution to Question #3(b) below

Additional space for Question 3

Question 4. [20 marks]

(a) Prove that if u is the harmonic conjugate of v in a domain Ω and v is the harmonic conjugate of u in Ω, then u and v are constant functions. [**10**]

Write your solution to Question #4(a) below

(b) Find all singularities of the function

$$f(z) = \frac{\sinh(z^3) - z}{z^5},$$

and determine the nature of each of these singularities (e.g. removable singularity, pole, essential singularity). [**10**]

Write your solution to Question #4(b) below

Additional space for Question 4

Question 5. [20 marks]

(a) State the Residue Theorem. [**10**]

Write your solution to Question #5(a) below

(b) Using the Residue Theorem, or otherwise, compute

$$\int_C \frac{z+1}{(z-1)(z+4)^3}\,\mathrm{dz},$$

where C is the positively oriented circle of radius 2 centred at $z = -3$. [**10**]

Write your solution to Question #5(b) below

Additional space for Question 5

This page is for additional work and will NOT be marked.

End of Paper.