Queen Mary
University of London

Main Examination period 2017

MTH5103
Complex Variables

Duration: 2 hours

Write your solutions in the space provided in this exam paper.

Student number: | | | | | | | | | |

Desk number: | | | |

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiners: M. Shamis

 Turn Over

This page is for marking purposes only:

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Question	Mark	Subpart Breakdown
1		
2		
3		
4		
5		
6		
TOTAL :		

Question 1. [15 marks]

(a) Find all solutions $z \in \mathbb{C}$ of the equation $z^4 = 1 - i$. **[5]**

Write your solution to Question #1(a) below

(b) What is the image of the line $\{z = t + (1 - t)i \mid \quad t \in \mathbb{R} \cup \{\infty\}\}$ under the transformation $z \to 1/z = w$? Provide the equation for the image and sketch the line in the z-plane and its image in the w-plane. **[5]**

Write your solution to Question #1(b) below

(c) Let the function $f(z)$ be defined on the set of non-zero complex numbers by the formula $f(z) = z/\bar{z}$. Show that f is not differentiable anywhere. [**5**]

Write your solution to Question #1(c) below

Additional space for Question 1

Question 2. [15 marks]

(a) State the Ratio Test. **[5]**

Write your solution to Question #2(a) below

(b) Using the Ratio Test, or otherwise, determine the values of z for which the power series

$$\sum_{n=1}^{\infty} \left(\frac{z}{i\,n}\right)^n$$

converges. What is the radius of convergence? **[10]**

Write your solution to Question #2(b) below

Question 3. [20 marks] Consider the function $f(z) = \dfrac{z-7}{z^2+z-2}$.

(a) Find the coefficients A, B, a, and b so that the function $f(z)$ has the following representation:

$$f(z) = \frac{A}{z-a} + \frac{B}{z-b}.$$

[1]

Write your solution to Question #3(a) below

(b) Using part (a), find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$$

of $f(z)$ on the annulus $1 < |z| < 2$. **[14]**

Write your solution to Question #3(b) below

(c) Determine the residue of $f(z)$ at the point $z = 1$. **[5]**

Write your solution to Question #3(c) below

Additional space for Question 3

Question 4. [15 marks]

(a) Suppose f is a complex function. Define what is meant by an **isolated singularity** of f. Define what is meant by an **essential singularity** of f.

[5]

Write your solution to Question #4(a) below

(b) Find all singularities of the function

$$f(z) = \frac{\sin(z-1)}{z^2 + 2z - 3},$$

and determine the nature of each of these singularities. **[10]**

Write your solution to Question #4(b) below

Additional space for Question 4

Question 5. [15 marks]

(a) State Rouché's Theorem. [**5**]

Write your solution to Question #5(a) below

(b) How many zeros (counted with multiplicity) does the polynomial

$$f(z) = 4z^4 - 29z^2 + 5$$

have in the annulus $2 < |z| < 4$? Justify your answer. **[10]**

Write your solution to Questions #5(b) below

Additional space for Question 5

Question 6. [20 marks]

(a) State the Residue Theorem. [**5**]

Write your solution to Question #6(a) below

(b) Using the Residue Theorem, or otherwise, compute

$$\int_C \frac{z+1}{(z-1)(z+2)^2}\,dz,$$

where C is the positively oriented circle of radius 5 centred at the origin. **[15]**

Write your solution to Question #6(b) below

Additional space for Question 6

This page is for additional work and will NOT be marked.

End of Paper.