Queen Mary
University of London

Main Examination period 2017

MTH4110/MTH4210: Mathematical Structures

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Examiners: O. F. Bandtlow, T. Popiel

Question 1. [12 marks]

(a) Explain what is meant by a **prime number**. **[4]**

(b) Prove that there are infinitely many prime numbers. **[8]**

Question 2. [12 marks] Let A, B and C be sets.

(a) Define the following sets:

(i) $A \cup B$, (ii) $A \setminus B$, (iii) $A \triangle B$. **[4]**

(b) Consider the following equalities.

(i) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

(ii) $(A \cup B) \setminus C = (A \setminus C) \cap (B \setminus C)$.

For each of them decide whether it is true or false. If it is true prove it without appealing to Venn diagrams; if it is false give a counterexample. **[8]**

Question 3. [16 marks]

(a) Let R be a relation on a set A. Explain what is meant by saying that R is

(i) **reflexive**, (ii) **symmetric**, (iii) **transitive**. **[6]**

(b) Give an example of a relation on $\mathbb{Z}$ which is transitive, but neither reflexive nor symmetric. **[2]**

(c) Let a relation R be defined on $\mathbb{C}$ by $a\,R\,b$ if and only if $|a| = |b|$. Show that R is an equivalence relation and describe the corresponding equivalence classes. **[8]**

Question 4. [12 marks] Let $f : \mathbb{N} \to \mathbb{N}$ be a function. Suppose that $f(k+l) = f(k) \cdot f(l)$ for all natural numbers k and l. Prove, using induction, that $f(n) = f(1)^n$ for all $n \in \mathbb{N}$. **[12]**

Question 5. [12 marks] Let a and b be integers.

(a) Explain what is meant by the **greatest common divisor of** a **and** b. **[4]**

(b) Suppose that $a = bq + r$ for some integers q and r. Show that $\gcd(a,b) = \gcd(b,r)$. **[4]**

(c) Calculate $\gcd(84, 60)$ using Euclid's algorithm. **[4]**

Question 6. [16 marks]

(a) Let

$$z = \frac{1}{\sqrt{2}}(1 - \mathrm{i}).$$

Determine the modulus and argument of z. Hence, or otherwise, find the real and imaginary parts of z^{2017}. **[6]**

(b) State the Fundamental Theorem of Algebra. **[4]**

(c) (i) Is every non-constant polynomial function $p : \mathbb{R} \to \mathbb{R}$ surjective?

(ii) Is every non-constant polynomial function $p : \mathbb{C} \to \mathbb{C}$ surjective?

In each case, give reasons for your answers. **[6]**

Question 7. [12 marks] Let x and y be real numbers. Consider the following statement.

If xy is irrational, then x or y is irrational.

(a) Write down the contrapositive. **[3]**

(b) Write down the converse. **[3]**

(c) Is the statement true? Is the contrapositive true? Is the converse true? Give reasons for your answers. **[6]**

Question 8. [8 marks]

(a) Find the flaw in the following proof: **[4]**

Theorem. Let a, b, c and d be positive real numbers, with $a/b < c/d$. Then

$$\frac{a}{b} < \frac{a+c}{b+d}.$$

Proof.

If $$\frac{a}{b} < \frac{a+c}{b+d},$$

then $$a(b+d) < (a+c)b,$$

so $$ab + ad < ab + cb,$$

hence $$\frac{a}{b} < \frac{c}{d},$$

and the assertion follows. □

(b) How can it be fixed? **[4]**

End of Paper.