Queen Mary
University of London

Main Examination period 2018

MTH4107 / MTH4207: Introduction to Probability

Duration: 2 hours

Student number | | | | | | | | | | **Desk number** | | | |

Make and model of calculator used _____

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more paper, ask an invigilator for an additional booklet and attach it to this paper at the end of the exam.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

This is an OPEN BOOK exam

permitted:	any printed material, e.g. books any handwritten notes photocopies of any kind
prohibited:	using communication devices, e.g. laptops or mobile phones sharing material with other students

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL.

If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it shall be treated as an assessment offence. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: W. Just

This page is for marking purposes only.

Do not write on it.

Question	Mark	Comments
1	/ 25	
2	/ 25	
3	/ 25	
4	/ 25	
Total		

Question 1. [25 marks] You have a fair and a biased coin. You toss the fair coin once. If it shows head you toss the fair coin again one time. If it shows tail you toss the biased coin one time.

(a) Write down the sample space for this experiment. **[5]**

(b) State the following events as subsets of the sample space: **[6]**

A: Both tosses produce the same result

B: I toss the same coin twice

C: I toss two different coins

(c) The biased coin has probability $2/3$ of showing head. Compute the probabilities of the events stated in part (b). **[6]**

(d) Compute the conditional probability of the event B given the event A. **[4]**

(e) Are the events B and C independent? Give a reason for your answer. [**4**]

Question 2. [25 marks] A bag contains 2 red balls, 3 red cubes, 3 blue balls, and 2 blue cubes. You pick 3 objects at random without replacement.

(a) Compute the probability that you draw 2 blue balls. Simplify the result and state the result as a fraction. **[6]**

(b) Compute the probability that all the objects you draw are blue. Simplify the result and state the result as a fraction. **[6]**

(c) Denote by C the random variable counting the number of cubes you draw. Compute the probability mass function of C. **[4]**

(d) Denote by R the random variable counting the number of red objects you draw. Compute the joint probability that $R = 0$ and $C = 3$. **[4]**

(e) State with reason whether the random variables R and C are independent. **[5]**

Question 3. [25 marks] A bus operator operates two lines each connecting the city Xinji with Yiwu. Line 1 runs a bus A from Xinji to Ulanqab and a bus B from Ulanqab to Yiwu, so that passengers have to change at Ulanqab. Line 2 runs a bus C from Xinji to Weihui and a bus D from Weihui to Yiwu, so that passengers have to change at Weihui. Buses are running with probabilities $\mathbb{P}(A) = 0.9$, $\mathbb{P}(B) = 0.8$, $\mathbb{P}(C) = 0.7$, $\mathbb{P}(D) = 0.8$. Furthermore the company ensures that at least three of the four buses are running.

(a) Using the inclusion-exclusion principle, or otherwise, compute the probability that I can travel via line 1 from Xinji to Yiwu. **[5]**

(b) Using the inclusion-exclusion principle, or otherwise, compute the probability that I can travel via line 2 from Xinji to Yiwu. **[5]**

(c) Assuming that I take bus A. Compute the probability that I can take bus B when changing at Ulanqab. **[5]**

(d) Assume that I take bus C. Compute the probability that I can take bus D when changing at Weihui. **[5]**

(e) Assume both buses, A and C are available at station Xinji. Which bus should I take to have a higher probability to reach the destination Yiwu. Justify your choice by comparing the relevant probabilities. **[5]**

Question 4. [25 marks] Let X be a random variable with $X \sim \text{Geom}(1/3)$. Let Y be another random variable which has Binomial$(N, 1/4)$ distribution where N is the value taken by the random variable X.

(a) State the expectation value of the random variable X. **[4]**

(b) Assuming X takes the value $X = N$ compute the conditional expectation value of the random variable Y. **[6]**

(c) Using the law of total probability, or otherwise, compute the expectation value of Y. **[6]**

(d) Using the law of total probability, or otherwise, compute the expectation value of the product XY and hence compute the covariance of the random variables X and Y. **[5]**

(e) Are the random variables independent ? Give a reason. **[4]**

Additional space for answers

Write your solutions here

Write your solutions here

Write your solutions here

Write your solutions here

Write your solutions here

End of Paper.