Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH 4106: Introduction to Statistics

Duration: 2 hours

Date and time: 29 April 2015, 10.00-12.00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. Please state on your answer book the name and type of machine used.
Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.
The New Cambridge Statistical Tables are provided.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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Exam papers must not be removed from the examination room.

Examiner(s): D. S. Coad and L. I. Pettit

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Question 1 (15 marks). The temperature, measured in degrees Celsius, of 25 patients with 'flu-like symptoms is given below.

37.9	38.4	38.7	39.5	39.0
39.5	38.1	37.5	37.8	39.2
37.4	38.6	38.2	38.9	37.9
39.3	38.7	38.5	38.8	38.8
37.6	39.4	38.9	38.2	38.3

(a) Draw a stem-and-leaf plot of the data. [**3**]

(b) Find the five-number summary for these data. [**5**]

(c) Comment on the distribution of the data in the light of your answers to (a) and (b). [**2**]

(d) For these data, the sample mean is 38.52 and the sample variance is 0.391. For a temperature of x measured in degrees Celsius, the corresponding temperature measured in degrees Fahrenheit is $1.8x + 32$. Find the mean and variance of the temperatures in degrees Fahrenheit, justifying your answers. [**5**]

Question 2 (15 marks). Let X and Y be random variables all of whose values are non-negative integers.

(a) Define the *probability generating function* of X. [**2**]

(b) State a theorem linking the probability generating functions of X, Y and $X+Y$ under a suitable condition. [**3**]

(c) Let X have a binomial distribution with $n = 20$ and $p = 0.3$, which will be written as $X \sim \mathrm{Bin}(20, 0.3)$. Derive the probability generating function of X. [**5**]

(d) Suppose that $X \sim \mathrm{Bin}(20, 0.3)$, that $Y \sim \mathrm{Bin}(30, 0.3)$, and that X and Y are independent of each other. Find the distribution of $X + Y$. [**5**]

Question 3 (10 marks). Let $X_1, \ldots, X_n$ be mutually independent random variables which all have the same distribution with expectation μ and variance σ^2. Let $\bar{X}$ be the sample mean.

(a) Prove that $\mathbb{E}(\bar{X}) = \mu$. [**3**]

(b) Find the variance of $\bar{X}$. [**4**]

(c) State the distribution of $\bar{X}$ when $X_i \sim N(\mu, \sigma^2)$ for $i = 1, 2, \ldots, n$. [**3**]

Question 4 (20 marks). (a) Let $X \sim U(0, 9)$ and put $Y = \sqrt{X}$. Find the probability density function of Y. Hence, find the expectation of Y. [**10**]

(b) Let $U \sim \mathrm{Exp}(3)$ and put $V = 2U + 6$. Find the probability density function of V. Find the expectation of V. [**10**]

Question 5 (5 marks). If $0.1178, 0.7843, 0.8713, 0.5472, 0.6977$ are five simulated values from a $U[0,1]$ distribution, simulate five values from a $\text{Bin}(7, 0.3)$ distribution.

Question 6 (20 marks). Consider the population of all people who are ordinarily resident in the London borough of Tower Hamlets. Let p be the proportion of these who have ever visited the USA. A travel agent wants to estimate p, so she chooses a random sample of 480 residents and asks each one if they have ever visited the USA. Let X be the number who answer "yes".

(a) State the distribution of X. Hence, write down its expectation and variance. [**4**]

(b) Find $\mathbb{P}(X \leq 90)$ if $p = 1/5$. [**6**]

(c) Let $Y = X/480$. Show that Y is an unbiased estimator of p. [**4**]

(d) Find the mean squared error of Y as a function of p. [**4**]

(e) Suppose that 106 residents in the sample answer "yes". Estimate the value of p, giving your answer to three decimal places. [**2**]

Question 7 (15 marks). A manufacturer of soft drinks sells them in bottles which are labelled as containing 500 ml each. A new manager thinks that the firm is putting slightly too much drink into each bottle, and therefore spending too much money. He assumes that the distribution of the volume, in ml, of drink in the bottles is $N(\mu, 4)$, and thinks that, if $\mu > 505$, then the firm should adjust the machines that fill the bottles. The manager takes a random sample of ten filled bottles and measures the volume, in ml, of drink that they contain as follows:

503	507	506	507	510
506	509	505	508	506

(a) State the manager's null and alternative hypotheses. [**3**]

(b) Find the sample mean and the sample standard deviation of the data. [**4**]

(c) Carry out the appropriate hypothesis test at the 5% significance level and report the conclusion. [**8**]

End of Paper.