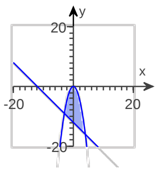
Student: ____________________	**Instructor:** Rainer Klages	**Assignment:** Semester B final assessment 2020/21
Date: ____________________	**Course:** MTH4101/MTH4201 Calculus II 2021	

1. Sketch the region bounded by the given line and curve. Then express the region's area as an iterated double integral and evaluate the integral.

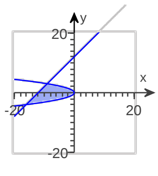
The parabola $x=-y^2$ and the line $y=x+12$

Sketch the region. Choose the correct graph below.

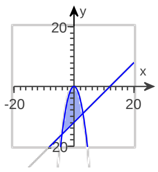
A.



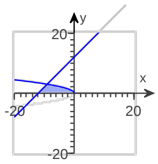
B.



C.



D.



Express the shaded area as an iterated integral. Select the correct choice below and fill in the answer boxes to complete your choice.

A. $A=\int_{0}^{-9}\int_{\_\_\_}^{\_\_\_} \_\_\_\_\_\_ \,dx\,dy+\int_{\_\_\_}^{\_\_\_}\int_{\_\_\_}^{\_\_\_} \_\_\_\_\_\_ \,dx\,dy$

B. $A=\int_{\_\_\_}^{\_\_\_}\int_{\_\_\_}^{\_\_\_} dx\,dy$

The value of the integral is ________. (Simplify your answer. Type an integer or a fraction.)

ID: 14.3.3

2. Find the Maclaurin series for the function.

$f(x)=8\,\mathbf{sin}\,2x$

$\sum_{n=0}^{\infty}$ ________ (Type an expression using n and x as the variables.)

ID: 9.8.15

3. Find the Taylor polynomial of order 3 generated by f at a.

$f(x)=\frac{1}{x+3},\ a=1$

A. $P_3(x)=\frac{1}{2}-\frac{x+1}{4}+\frac{(x+1)^2}{8}-\frac{(x+1)^3}{16}$

B. $P_3(x)=\frac{1}{4}-\frac{x-1}{16}+\frac{(x-1)^2}{64}-\frac{(x-1)^3}{256}$

C. $P_3(x)=\frac{1}{4}-\frac{x+1}{16}+\frac{(x+1)^2}{64}-\frac{(x+1)^3}{256}$

D. $P_3(x)=\frac{1}{2}-\frac{x-1}{4}+\frac{(x-1)^2}{8}-\frac{(x-1)^3}{16}$

ID: 9.8-2

4. Use any method to determine if the series converges or diverges. Give reasons for your answer.

$$\sum_{n=1}^{\infty}\frac{n^{\sqrt{2}}}{16^n}$$

Select the correct choice below and fill in the answer box to complete your choice.

A. The series converges because the limit found using the Ratio Test is ________.

B. The series diverges because the limit found using the Ratio Test is ________.

C. The series converges because the limit used in the nth-Term Test is ________.

D. The series diverges because the limit used in the nth-Term Test is ________.

ID: 9.5.17

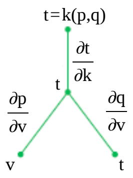
5. Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3}{\sqrt{n}}-\frac{3}{\sqrt{n+1}}\right)$.

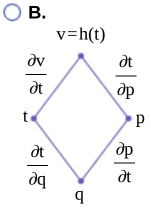
The sum of the series is ____________.
(Type an exact answer, using radicals as needed.)

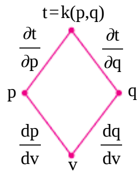
ID: 9.2.49

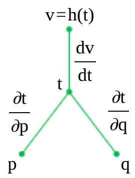
6. Draw a dependency diagram, and write a chain rule formula for $\frac{\partial v}{\partial p}$ and $\frac{\partial v}{\partial q}$, where $v = h(t)$ and $t = k(p,q)$.

Choose the correct dependency diagram below.

A.


B.


C.


D.


Choose the correct chain rule formula for $\frac{\partial v}{\partial p}$ below.

A. $\frac{\partial v}{\partial p}=\frac{\partial t}{\partial p}\frac{\partial t}{\partial q}-\frac{dv}{dt}$

B. $\frac{\partial v}{\partial p}=\frac{dv}{dt}\frac{\partial t}{\partial p}$

C. $\frac{\partial v}{\partial p}=\frac{\partial t}{\partial p}\frac{\partial t}{\partial q}+\frac{dv}{dt}$

D. $\frac{\partial v}{\partial p}=\frac{dv}{dt}\frac{\partial t}{\partial q}$

Choose the correct chain rule formula for $\frac{\partial v}{\partial q}$ below.

A. $\frac{\partial v}{\partial q}=\frac{\partial t}{\partial p}\frac{\partial t}{\partial q}+\frac{dv}{dt}$

B. $\frac{\partial v}{\partial q}=\frac{\partial t}{\partial p}\frac{\partial t}{\partial q}-\frac{dv}{dt}$

C. $\frac{\partial v}{\partial q}=\frac{dv}{dt}\frac{\partial t}{\partial p}$

D. $\frac{\partial v}{\partial q}=\frac{dv}{dt}\frac{\partial t}{\partial q}$

ID: 13.4.21

7.

Given the function f(x,y) = xy, answer the following questions.
a. Find the function's domain.
b. Find the function's range.
c. Describe the function's level curves.
d. Find the boundary of the function's domain.
e. Determine if the domain is an open region, a closed region, both, or neither.
f. Decide if the domain is bounded or unbounded.

a. Choose the correct domain of the function f(x,y) = xy.

- **A.** All points in the first quadrant
- **B.** All points in the xy-plane except the origin
- **C.** All points in the xy-plane
- **D.** $y \ge x$

b. Choose the correct range of the function f(x,y) = xy.

- **A.** All non-negative real numbers
- **B.** All non-negative integers
- **C.** All real numbers
- **D.** All integers

c. Choose the correct description(s) of the level curves of f(x,y) = xy. Select all that apply.

- **A.** Straight lines, when $f(x,y) \ne 0$
- **B.** Hyperbolas, when $f(x,y) \ne 0$
- **C.** The x- and y-axes, when $f(x,y) = 0$
- **D.** Circles, when $f(x,y) \ne 0$

d. Does the function's domain have a boundary? Select the correct choice and if necessary, fill in the answer box below to complete your choice.

- **A.** Yes, at ________ = 0
  (Type an expression using x and y as the variables.)
- **B.** Yes, at ________
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **C.** No

e. Choose the correct description of the domain of f(x,y) = xy.

- Both open and closed
- Open region
- Neither open nor closed
- Closed region

f. Is the domain of f(x,y) = xy bounded or unbounded?

- Unbounded
- Bounded

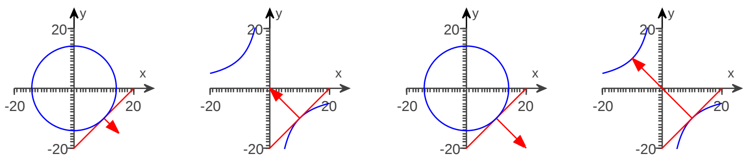
ID: 13.1.21

8. Sketch the curve f(x,y) = c together with ∇f and the tangent line at the given point. Then write an equation for the tangent line.

$xy = -100, \ (10, -10)$

Choose the correct graph below.

A. **B.** **C.** **D.**



The equation for the tangent line is ________.
(Type an equation using x and y as the variables.)

ID: 13.5.27

9. By considering different paths of approach, show that the function below has no limit as $(x,y) \to (0,0)$.

$$h(x,y) = \frac{x^2 + y}{y}$$

Examine the values of h along curves that end at (0,0). Along which set of curves is h a constant value?

- **A.** $y = kx + kx^2,\ x \neq 0,\ k \neq 0$
- **B.** $y = kx^2,\ x \neq 0,\ k \neq 0$
- **C.** $y = kx,\ x \neq 0,\ k \neq 0$
- **D.** $y = kx^3,\ x \neq 0,\ k \neq 0$

If (x,y) approaches (0,0) along the curve when $k = 1$ used in the set of curves found above, what is the limit?

__________ (Simplify your answer.)

If (x,y) approaches (0,0) along the curve when $k = 2$ used in the set of curves found above, what is the limit?

__________ (Simplify your answer.)

What can you conclude?

- **A.** Since f has the same limit along two different paths to (0,0), it cannot be determined whether or not f has a limit as (x,y) approaches (0,0).
- **B.** Since f has two different limits along two different paths to (0,0), by the two-path test, f has no limit as (x,y) approaches (0,0).
- **C.** Since f has the same limit along two different paths to (0,0), by the two-path test, f has no limit as (x,y) approaches (0,0).
- **D.** Since f has two different limits along two different paths to (0,0), it cannot be determined whether or not f has a limit as (x,y) approaches (0,0).

ID: 13.2.47

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10. Reverse the order of integration and then evaluate the integral.

$$\int_0^{18} \int_{y/3}^{6} e^{x^2}\, dx\, dy$$

- **A.** $\frac{3}{2}\left(e^{36} - 1\right)$
- **B.** $\frac{3}{4} e^{36}$
- **C.** $\frac{3}{4}\left(e^{36} - 1\right)$
- **D.** $\frac{3}{2} e^{36}$

ID: 14.2-18

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11. Find out whether the series given below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\ln(n+6)}{n+6}$$

Choose the correct answer below.

- **A.** The integral test shows that the series converges.
- **B.** The nth-term test shows that the series converges.
- **C.** The integral test shows that the series diverges.

ID: 9.3.21

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12. Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(\ln n)^5}{\sqrt{n}}$$

- **A.** The sequence converges to 0.
- **B.** The sequence converges to $\ln 5$.
- **C.** The sequence converges to e^5.
- **D.** The sequence diverges.

ID: 9.1-25

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13. Find the directions in which the function increases and decreases most rapidly at P_0. Then find the derivatives of the function in these directions.

$f(x,y,z) = (x/y) - yz, \quad P_0(-3,-1,4)$

The direction in which the given function $f(x,y,z) = (x/y) - yz$ increases most rapidly at $P_0(-3,-1,4)$ is **u** = ____ **i** + (____) **j** + (____) **k**.
(Type exact answers, using radicals as needed.)

The direction in which the given function $f(x,y,z) = (x/y) - yz$ decreases most rapidly at $P_0(-3,-1,4)$ is $-$**u** = ____ **i** + (____) **j** + (____) **k**.
(Type exact answers, using radicals as needed.)

The derivative of the given function $f(x,y,z) = (x/y) - yz$ in the direction in which the function increases most rapidly at $P_0(-3,-1,4)$ is $\left(D_{\mathbf{u}}f\right)_{(-3,-1,4)}$ = ____.
(Type an exact answer, using radicals as needed.)

The derivative of the given function $f(x,y,z) = (x/y) - yz$ in the direction in which the function decreases most rapidly at $P_0(-3,-1,4)$ is $\left(D_{-\mathbf{u}}f\right)_{(-3,-1,4)}$ = ____.
(Type an exact answer, using radicals as needed.)

ID: 13.5.21

14. Does the sequence $\{a_n\}$ converge or diverge? Find the limit if the sequence is convergent.

$a_n = 7 + (-1)^n$

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** The sequence converges to $\lim_{n\to\infty} a_n$ = ____.
  (Simplify your answer.)
- **B.** The sequence diverges.

ID: 9.1.39

15. For the series below, **(a)** find the series' radius and interval of convergence. For what values of x does the series converge **(b)** absolutely, **(c)** conditionally?

$$\sum_{n=1}^{\infty} \frac{36^n x^{2n}}{n}$$

(a) The radius of convergence is ____.
(Type an integer or a simplified fraction.)

Determine the interval of convergence. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The interval of convergence is ____.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges only at x = ____. (Type an integer or a simplified fraction.)
- **C.** The series converges for all values of x.

(b) For what values of x does the series converge absolutely?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges absolutely for ____.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges absolutely at x = ____. (Type an integer or a simplified fraction.)
- **C.** The series converges absolutely for all values of x.

(c) For what values of x does the series converge conditionally?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges conditionally for ____.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges conditionally at x = ____.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- **C.** There is no value of x for which the series converges conditionally.

ID: 9.7.13

16. Evaluate the following integral.

$$\int_0^2 \int_0^{4-4x} \int_0^{4-4x-y} dz\, dy\, dx$$

$\int_0^2 \int_0^{4-4x} \int_0^{4-4x-y} dz\, dy\, dx =$ ____________ (Simplify your answer.)

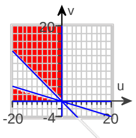
ID: 14.5.10

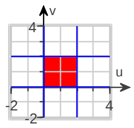
17.
a. Solve the system $u = 2x + 3y$, $v = x + 5y$ for x and y in terms of u and v. Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.
b. Find the image under the transformation $u = 2x + 3y$, $v = x + 5y$ of the triangular region in the xy-plane bounded by the x-axis, the y-axis, and the line $x + y = 2$. Sketch the transformed image in the uv-plane.

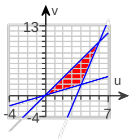
a. x = ____________ , y = ____________

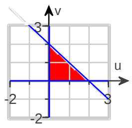
The Jacobian is ____________.

b. Choose the correct sketch of the transformed region in the uv-plane below.

A.


B.


C.


D.


ID: 14.8.3

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18. Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = x^2 + xy + 3x + 3y - 5$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local maximum occurs at ____________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local maximum value(s) is/are ____________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local minimum occurs at ____________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local minimum value(s) is/are ____________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A saddle point occurs at ____________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no saddle points.

ID: 13.7.3

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19. Find all the second order partial derivatives of $g(x,y) = x^5 y + 2 \sin(y) + 4y \cos(x)$.

$\frac{\partial^2 g}{\partial x^2} =$ ____________

$\frac{\partial^2 g}{\partial y \partial x} =$ ____________

$\frac{\partial^2 g}{\partial y^2} =$ ____________

$\frac{\partial^2 g}{\partial x \partial y} =$ ____________

ID: 13.3.43

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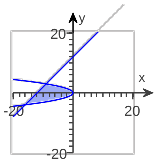
20. Evaluate the double integral over the given region R.

$$\iint_R \left(9y^2 - 6x\right) dA \quad R:\ 0 \le x \le 3,\ 0 \le y \le 1$$

$$\iint_R \left(9y^2 - 6x\right) dA = \_\_\_\_\_\_\_\_\_\_$$

ID: 14.1.17



1. B.

B. $A = \int_{-4}^{3} \int_{y-12}^{-y^2} dx\, dy$

$\frac{343}{6}$

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2. $\frac{(-1)^n \cdot 8 \cdot 2^{2n+1} x^{2n+1}}{(2n+1)!}$

3.
B. $P_3(x) = \frac{1}{4} - \frac{x-1}{16} + \frac{(x-1)^2}{64} - \frac{(x-1)^3}{256}$

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4. A. The series converges because the limit found using the Ratio Test is $\frac{1}{16}$.

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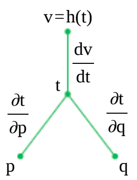
5. 3

6.

D. 

B. $\frac{\partial v}{\partial p} = \frac{dv}{dt}\frac{\partial t}{\partial p}$

D. $\frac{\partial v}{\partial q} = \frac{dv}{dt}\frac{\partial t}{\partial q}$

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7. C. All points in the xy-plane

C. All real numbers

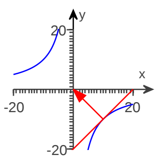
B. Hyperbolas, when $f(x,y) \neq 0$, C. The x- and y-axes, when $f(x,y) = 0$

C. No

Both open and closed

Unbounded

8.

B. 

$y = x - 20$

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9. B. $y = kx^2$, $x \neq 0$, $k \neq 0$

2

$\frac{3}{2}$

B. Since f has two different limits along two different paths to (0,0), by the two-path test, f has no limit as (x,y) approaches (0,0).

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10. A. $\frac{3}{2}\left(e^{36} - 1\right)$

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11. C. The integral test shows that the series diverges.

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12. A. The sequence converges to 0.

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13. $-\frac{1}{\sqrt{3}}$

$-\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$

$-\frac{1}{\sqrt{3}}$

$\sqrt{3}$

$-\sqrt{3}$

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14. B. The sequence diverges.

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15. $\frac{1}{6}$

A. The interval of convergence is $-\frac{1}{6} < x < \frac{1}{6}$.(Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)

A. The series converges absolutely for $-\frac{1}{6} < x < \frac{1}{6}$.(Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)

C. There is no value of x for which the series converges conditionally.

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16. $\frac{16}{3}$

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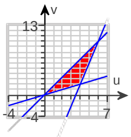
17. $\frac{5u - 3v}{7}$

$\frac{2v - u}{7}$

$\frac{1}{7}$

C. 

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18. B. There are no local maxima.

B. There are no local minima.

A. A saddle point occurs at **(−3,3)** .(Type an ordered pair. Use a comma to separate answers as needed.)

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19. $20x^3y - 4y\cos x$

$5x^4 - 4\sin x$

$-2\sin y$

$5x^4 - 4\sin x$

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20. -18
