MTH4101: Calculus II

Duration: 2 hours

Date and time: 17th May 2016, 14:30–16:30

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You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiner(s): R. J. Harris, J. A. Valiente Kroon

Question 1.

(a) Use the Sandwich Theorem for sequences to find

$$\lim_{n\to\infty} \frac{\cos n}{n}.$$ [7]

(b) Find the sum of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{3^{n-1}}.$$ [7]

(c) Find the Taylor polynomials of orders 0, 1, and 2 for the function

$$f(x) = \sin x,$$

about the point $x = \pi/4$. [7]

(d) Evaluate

$$\lim_{\substack{(x,y)\to(2,0)\\ 2x-y\neq 4}} \frac{2x-y-4}{\sqrt{2x-y}-2}.$$ [7]

(e) Find all first-order and second-order partial derivatives of the function

$$g(x,y) = y^2 e^{x^3}.$$ [7]

(f) Find the unit vector in the direction in which $f(x,y,z) = xe^y + z^2$ increases most rapidly at the point $(1, \ln 2, 1/2)$. [7]

(g) Evaluate the triple integral

$$\int_{-1}^{1}\int_{0}^{2}\int_{0}^{4} (x+y+z)\,dy\,dx\,dz.$$ [7]

(h) Solve the differential equation

$$\frac{dy}{dx} = (2+y)e^{3x}, \quad y > -2,$$

giving the solution in explicit form. [7]

Question 2.

(a) State the nth-Term Test for Divergence of infinite series. [**4**]

(b) Demonstrate the use of the test in part (a) for the series

$$\sum_{n=1}^{\infty} \frac{\ln(n+1)}{\sqrt{n}}$$

and explain what you can, or cannot, conclude. [**7**]

Question 3.

(a) Use the Chain Rule for partial differentiation to express $\partial z/\partial u$ and $\partial z/\partial v$ as functions of u and v for

$$z = 3xe^y, \quad x = u + v, \quad y = \ln(uv^2).$$ [**10**]

(b) Write a sentence to explain how you could check your result for part (a) without using the Chain Rule. [**1**]

Question 4. Consider a circle of radius two centred at the origin. Use the method of Lagrange multipliers to find the points on this circle where the function

$$f(x, y) = 3x - y + 5$$

has its extreme values. [**11**]

Question 5.

(a) Solve the system $u = 2x - 3y$, $v = -x + y$ to find expressions for x and y in terms of u and v. Use these expressions to find the Jacobian $\partial(x, y)/\partial(u, v)$. [**5**]

(b) Consider the integral

$$\int\int_R (x - y)^2 \, dx \, dy,$$

for the region R bounded by the lines $x = -6$, $x = 0$, $y = x$, and $y = x + 2$. Use the transformation from part (a) to rewrite this as an integral with respect to u and v and sketch the transformed region of integration in the uv-plane. [Evaluation of the integral is not required here.] [**6**]

End of Paper.