Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH4101: Calculus II

Duration: 2 hours

Date and time: 5th May 2015, 10:00–12:00

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You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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Examiner(s): R. Klages, Y. Fyodorov

Question 1.

(a) Obtain the limit as $n \to \infty$ for the sequence

$$a_n = \frac{\ln n}{n} \,.$$ **[7]**

(b) Use a suitable test to determine whether the series

$$\sum_{k=2}^{\infty} (-1)^k \frac{2}{3 \ln k}$$

converges or diverges. **[7]**

(c) Consider the function

$$f(x, y) = \frac{y^2 - 2xy + x^2}{x - y} \quad , \quad x \neq y \,.$$

Find the limit of f as $(x, y) \to (2, 2)$. **[7]**

(d) Find all first-order partial derivatives of the function

$$f(x, y) = 2x^{3y} \,.$$ **[7]**

(e) Find the directional derivative of the function

$$f(x, y, z) = \cos(yz)e^x \,,$$

at the point $(0, 0, 0)$, in the direction of the vector $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. **[7]**

(f) Find the area of the region enclosed by $y^2/2 = x$ and $2y = x$. **[7]**

(g) Evaluate the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \mathrm{d}x \, \mathrm{d}y$$

by transforming to polar coordinates. **[7]**

(h) Solve the differential equation

$$\frac{dy}{dx} = e^{x-y}$$

by giving the solution in implicit form. **[7]**

Question 2. Consider the function

$$f(x) = \frac{1}{x-1} \,.$$

(a) Find the Taylor series generated by f at $x = 2$. **[6]**

(b) Where, if anywhere, does the series converge to f? **[5]**

Question 3. Assume that $F(x, y) = x^3 - 2y^2 + xy = 0$ defines y as a differentiable function of x.

(a) Differentiate the whole equation with respect to x and then solve for dy/dx. **[4]**

(b) State the formula for implicit differentiation. **[3]**

(c) Use this formula to find dy/dx at the point $(-1, 1)$. **[4]**

Question 4. The surfaces

$$f(x, y, z) = x^2 + y^2 - 2 = 0 \quad \text{(cylinder)}$$

and

$$g(x, y, z) = x + z - 4 = 0 \quad \text{(plane)}$$

meet in an ellipse E. Find the parametric equation for the line tangent to E at the point $P_0(1, 1, 3)$. **[11]**

Question 5. Find all locations and values of the local maxima, local minima, and saddle points of the function

$$f(x, y) = \frac{x^3}{3} + 9y^3 - xy.$$

[11]

End of Paper.