Queen Mary
University of London

B. Sc. Examination by course unit 2014

MTH4100 Calculus 1

Duration: 2 hours

Date and time: 2 May 2014, 10:00h–12:00h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): Prof. B. Jackson

Question 1 (a) Find the set of all $x \in \mathbb{R}$ which satisfy the inequality

$$|x-1| \leq x^2 - 1.$$

[5 marks]

(b) Let

$$f(x) = \frac{x^2 - 4x + 4}{x^2 - x - 2}$$

for all $x \in \mathbb{R} \setminus \{-1, 2\}$. Determine whether each of the following limits exist, giving the value if it exists and a reason if it does not exist:

$$\lim_{x \to -1} f(x); \qquad \lim_{x \to 2} f(x); \qquad \lim_{x \to \infty} f(x).$$

Determine whether f has a continuous extension at $x = -1$ or $x = 2$, defining the extension if it exists and giving a reason if it does not exist. [7 marks]

(c) Define the function arctan, specifying both its domain and codomain, and determine its derivative. [7 marks]

(d) Find the equation of the tangent to the curve $y^3 + x^2 y - 3x + 1 = 0$ at the point $(1, 1)$. [5 marks]

(e) Find the area of the region bounded above by the curve $x^2 + y^2 = 2$ and below by the curve $y = x^2$.

[8 marks]

(f) Evaluate

$$\int (x+2) \ln(x-3)\, dx\,.$$

[8 marks]

Question 2 Consider the curve $y = f(x)$ for the function $f(x) = 3x^{\frac{2}{3}}(5 - x)$.

(a) Identify the domain of f and determine whether or not f is an even function or an odd function. [2 marks]

(b) Find $f'(x)$ and $f''(x)$. [4 marks]

(c) Find the critical points of f, determine where f is increasing or decreasing, and determine the behavior of f at each of its critical points. [7 marks]

(d) Find the inflexion points for f, if any occur, and determine the concavity of the curve. [5 marks]

(e) Determine the behavior of $f(x)$ as $x \to \pm\infty$ and identify any asymptotes. [2 marks]

(f) Plot key points, such as intercepts, critical points, and points of inflexion, and sketch the curve. [5 marks]

Question 3 (a) Differentiate $f(x) = \sin x$ from first principles. (You may assume the values for $\lim_{h\to 0} \frac{\sin h}{h}$ and $\lim_{h\to 0} \frac{\cos h - 1}{h}$ in your proof.) [5 marks]

(b) State, without proving, the Mean Value Theorem. [5 marks]

(c) Using the Mean Value Theorem, or otherwise, prove that if f is a differentiable function on an open interval I and $f'(x) > 0$ for all $x \in I$ then f is increasing on I. [5 marks]

Question 4 (a) Let f be a continuous function defined on an interval $[a, b]$ and

$$a = x_0 < x_1 < \ldots < x_n = b$$

be a partition of $[a, b]$.

(i) Define the *upper* and *lower Riemann sums* for f with respect to this partition. [5 marks]

(ii) Explain what it means to say that f is *integrable* on $[a, b]$. [5 marks]

(b) Calculate the upper Riemann sum for $f(x) = x^2$ on $[0, 1]$ with respect to the partition $x_0 < x_1 < \ldots < x_n$ when $x_i = i/n$ for all $0 \leq i \leq n$. Determine the limit of this sum as $n \to \infty$. [10 marks]

End of Paper