Queen Mary
University of London

Main Examination period 2023 – May/June – Semester B

MTH5131: Actuarial Statistics

Duration: 2 hours

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

Examiners: D. S. Stark, J. Griffin

Question 1 [10 marks]. A study took a random sample of people and examined their alcohol drinking habits. Each person was classified as either a light, moderate or heavy drinker. The researcher looked at the mean stress level of each group.

(a) Is this study observational or an experiment? Justify your answer. **[4]**

(b) Is this study descriptive, inferential, or predictive. Justify your answer. **[3]**

(c) Suppose that no information about the light drinkers was recorded. What do we call this kind of data? **[3]**

Question 2 [12 marks].

(a) Let $\{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ be an orthonormal basis for $\mathbb{R}^n$, and let $\lambda_1, \ldots, \lambda_n$ be any real scalers. Define the matrix

$$A = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^T + \cdots + \lambda_n \mathbf{u}_n \mathbf{u}_n^T$$

where T denotes transpose. Show that $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A. **[7]**

(b) Suppose that a data matrix of two observations of two variables is

$$B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

Find the components of B. **[5]**

Question 3 [17 marks]. Six students were ranked by their A Level Mathematics marks and then by their Calculus I marks at Queen Mary. The rankings of the students were

Student	A Level	Calculus I
1	4	4
2	3	3
3	2	1
4	6	5
5	5	2
6	1	6

(a) (i) Compute the Spearman correlation coefficient r_s between the two rankings. **[5]**

(ii) For a t-distribution approximation involving r_s, find the p-value of the t-statistic and use it to decide whether or not the rankings are correlated at 5% confidence level. **[5]**

(b) Compute the Kendall correlation coefficient between the two rankings. **[7]**

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Question 4 [8 marks]. Let $Y_1, \ldots, Y_n$ be independent $\text{Bin}(m, \pi)$ random variables, where m is known. Consider the estimator for π

$$T_n = \frac{1}{(n+1)m} \sum_{i=1}^{n} Y_i.$$

(a) Find the Mean Square Error of the estimators T_n. **[6]**

(b) Show that the sequence of estimators T_n is consistent. **[2]**

Question 5 [19 marks]. Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed with p.d.f.

$$f_Y(y) = \begin{cases} \frac{\theta 2^\theta}{y^{\theta+1}} & \text{for } y \geq 2, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 1$ is a parameter.

(a) Find the Cramér-Rao lower bound of the variance of unbiased estimators of

$$\frac{\theta}{\theta - 1}.$$

[7]

Suppose that the revenue in thousands of pounds produced by five of a companies clients are 3, 4, 3, 2, 10. It is assumed that revenue from the clients $Y_1 . \ldots , Y_5$ have the distribution given above.

(b) Find the method of moments estimate of θ. **[5]**

(c) Find the maximum likelihood estimate of θ. **[7]**

Question 6 [8 marks]. A factory produces widgets on an assembly line. After they are produced, the widgets are inspected to see if they are faulty. Widgets are faulty with probability θ independently of each other. The prior distribution of θ is $\text{Beta}(1, 19)$. Suppose that at the start of inspection the first 29 widgets are not faulty, but the 30th widget is faulty.

(a) Given this information, what is the posterior distribution of θ? **[6]**

(b) Find the Bayesian estimate of θ under quadratic loss. **[2]**

Question 7 [13 marks]. Suppose the number of claims arising from a risk each year, has a Poisson(λ) distribution. The prior distribution for λ is Gamma$(340, 2)$. You have been given data as shown in the first two columns in the table below.

Year	Pure Premium	Credibility factor at the start of year	Average number of claims based on number of years of past data available at the start of year	At the start of the year, the credibility estimate of the number of claims in the coming year
1	160			
2	185			
3	150			

Using the data from the first two columns, calculate the remaining columns. **[13]**

Question 8 [13 marks]. A study considered 248 Canadian firms. The number of interlocks (persons sitting on the boards of at least one other firm in the data set) was the response variable. A GLM Poisson model with log link was used. The covariates were their assets (measured in billions of dollars); nation of control (Canada, the United States, the United Kingdom, or another country); and the principal sector of operation of the firm (10 categories, including banking, other financial institutions, heavy manufacturing, etc.). Backward selection was used. The following scaled deviances were found for models containing the indicated covariates.

Covariates	Deviance
Assets, Nation, Sector	1887.402
Nation, Sector	2278.298
Assets, Sector	2216.345
Assets, Nation	2248.861

(a) Find the degrees of freedom for all of the models. **[6]**

(b) Use the χ^2 test to determine which of the covariates are significant in the largest model. **[7]**

End of Paper.