Queen Mary
University of London

**Main Examination period 2021 – January – Semester A**

# MTH6157: Survival Models

**You should attempt ALL questions. Marks available are shown next to the questions.**

**In completing this assessment:**

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

**IFoA exemptions.** For actuarial students, this module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, **you must submit within the first 3 hours of the assessment period**.

**Examiners: C. Sutton, M. Poplavskyi**

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**Question 1 [23 marks].** A university introduces a new actuarial science module which has a one hour online lecture each week for 10 weeks with an online quiz at the end of each lecture. Students must attend the lecture and pass the quiz to be allowed to remain on the module for the next week. There is no final exam for this module, instead the university will pass with full marks those students who successfully complete a certain number of quizzes. To set the number of weeks for a pass the university trials this module with a group of 20 students. The table below shows how many students fail the quiz each week and how many who are eligible do not log on to the online lecture.

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Do not log on | 0 | 1 | 0 | 0 | 0 | 3 | 0 | 1 | 0 | 0 |
| Fail the quiz | 1 | 1 | 0 | 1 | 2 | 1 | 2 | 2 | 0 | 1 |

(a) If the university would like a 65% pass rate for this module, use a Kaplan Meier estimate to calculate the number of quizzes students should complete to pass this module. Define carefully all the terms you use in the calculation. [**12**]

(b) What assumptions have you made in your calculations in (a) above? [**6**]

(c) What concerns should the module organiser have about this study being used to set the pass criteria for future years? [**5**]

**Question 2 [18 marks].** A trial is conducted for a new drug designed as a treatment for people who test positive for a respiratory virus which they have already suffered from once before. Results are analysed using a Cox's Proportional Hazard Model and the model fitted is

$$\lambda(t) = \lambda_0(t) \cdot \exp(\beta \cdot z^T)$$

where $\lambda(t)$ is the hazard at time $t$ months since administering the drug, and $\lambda_0(t)$ is the baseline hazard.

$z$ is a vector of covariates where $z_1$ measures the number of other pre-existing lung conditions the patient has; $z_2$ is the period in months between the two positive tests for this virus minus 2; and $z_3$ is 1 if the patient is age 65 or over and 0 otherwise.

$\beta$ is a vector of parameters where $\beta_1 = 0.05$; $\beta_2 = 0.1$; $\beta_3 =$ -0.3

(a) State the group of lives to whom the baseline hazard applies. **[3]**

(b) For an 85-year-old patient suffering from one other lung condition who previously tested positive 7 months ago,
(i) Write down the hazard function in terms of the baseline hazard.
(ii) Express the survival function in terms of the baseline hazard. **[5]**

(c) A 47-year-old with no pre-existing health conditions is given the drug when they test positive for the virus 3 months after a previous positive test. If they have a 93% probability of surviving 4 months following a second positive test, calculate the probability that the patient in (b) above will survive 4 months. **[10]**

**Question 3 [21 marks].** The 92-series mortality tables published by the Continuous Mortality Investigation (CMI) at the Institute and Faculty of Actuaries uses a Poisson survival model to estimate the force of mortality at each age from 17 to 120. Different tables are produced for male / female lives and by smoker / non-smoker status.

(a) If a sample of N male non-smokers all age x are monitored for a year with d deaths counted in the year and a total of V years of waiting time measured, derive from first principles an expression for the maximum likelihood estimate of the force of mortality in this model. **[9]**

(b) What assumptions have been made in using the Poisson model and the estimate in (a)? **[4]**

(c) At which parts of the age range are these assumptions more problematic and why? **[3]**

(d) An insurance company that offers health and life insurance would like to take this CMI model and extend it for use in its premium calculations. What issues does the company face and how might these be overcome? **[5]**

**Question 4 [15 marks].** The guard, sometimes called the train manager, on a passenger train travelling from London to Manchester has two tasks. The first is to count the total number of passengers who get on and off the train at each station, and the second is to issue penalty fines of £50 to each passenger without a valid ticket. The table below gives the time the train stops at each station and the number of passengers counted.

| Time | Station | Passengers getting on | Passengers getting off |
|---|---|---|---|
| 09:20 | London | 139 | 0 |
| 09:50 | Milton Keynes | 33 | 29 |
| 10:48 | Stoke-on-Trent | 17 | 3 |
| 11:16 | Stockport | 7 | 2 |
| 11:27 | Manchester | 0 | 162 |

(a) On arrival at Manchester the guard has collected £200 in penalty fines. Calculate the rate of issuing penalties per person hour on this train journey. [**12**]

(b) What assumptions have you made in this calculation? [**3**]

**Question 5 [23 marks].** An insurance company with a large established annuity business has noticed an increase in early retirements from people in their late 50's since the beginning of the COVID-19 pandemic. It wishes to check that the standard table it uses for mortality rates still applies to that age group. It collects data on recent experience from existing annuitants as set out in the table below.

| *Age* $x$ | Observed number of deaths | Central Exposed to to Risk (policy years) | Standard table mortality rate |
|---|---|---|---|
| 53 | 4 | 1387 | 0.0024 |
| 54 | 4 | 1348 | 0.0028 |
| 55 | 4 | 1304 | 0.0032 |
| 56 | 5 | 1294 | 0.0037 |
| 57 | 7 | 1283 | 0.0043 |
| 58 | 6 | 1263 | 0.0050 |
| 59 | 9 | 1238 | 0.0058 |
| 60 | 9 | 1203 | 0.0068 |

(a) Perform a test of the overall goodness-of-fit of the observed data to the standard table at a 95% confidence interval stating any assumptions that you make. [**9**]

(b) Why might this test alone be insufficient when checking the suitability of the standard table in light of the COVID-19 pandemic? [**4**]

(c) Suggest two further tests that should be performed with reasons. [**4**]

(d) Why might the insurance company be particularly concerned about changing mortality rates amongst this particular group of new customers? [**6**]

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**End of Paper.**