Queen Mary
University of London

**Main Examination period 2020 – January – Semester A**

# MTH6157 / MTH6157P: Survival Models

**Duration: 2 hours**

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**You should attempt ALL questions. Marks available are shown next to the questions.**

The New Cambridge Statistical Tables are provided.
**Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.**

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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**Examiners: C. Sutton, M. Poplavskyi**

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**Question 1 [21 marks].** A popular grocery shop allows people to pay for their shopping either at a checkout counter with a shop employee or at a self-service machine where the shopper scans items themselves. The shop has two queues for people waiting to pay, one queue for the checkout counter and the other queue for the self-service machine. An actuarial student records the length of time some shoppers wait in these queues. The observed waiting times in each queue are shown below. People who leave a queue before reaching the checkout counter or self-service machine are marked with *.

Waiting times for the checkout counter in minutes:

8, 1, 1, 3, 6, 2*, 7*, 4, 3, 2, 6*, 5, 2, 3*, 2, 1

Waiting times for the self-service machine in minutes:

5, 3, 5, 3, 4, 2, 7, 4, 6, 7, 4, 3, 2, 8, 7*, 6

(a) Calculate the Kaplan-Meier estimates of the survival function for these two queues separately. [**14**]

(b) The actuarial student decides that they will use this analysis to determine which of the two queues they will join when paying in this shop. Which queue will they use and why? [**3**]

(c) A more experienced actuary tells the student their decision in (b) above does not allow properly for censoring. Explain why this might be the case. [**4**]

**Question 2 [21 marks].** A study is made into the mortality of patients admitted with pneumonia to two hospitals, one in New York and the other in Los Angeles. The results are analysed with a Cox Proportional Hazards Model and the model fitted is

$$h_i(t) = h_0(t)exp(\beta\ z^T)$$

where
$h_i(t)$ is the hazard at time $t$ days
$h_0(t)$ is the baseline hazard.
$z = (z_1, z_2, z_3)$ is a vector of covariates where $z_1$ is 1 in New York and 0 for patients in Los Angeles; $z_2$ is the time in hours after admission to hospital before antibiotics are administered; and $z_3$ is 1 if the patient is age 65 or over and 0 otherwise.
$\beta = (\beta_1, \beta_2, \beta_3)$ is a vector of parameters where $\beta_1 = 0.1$; $\beta_2 = 0.02$; $\beta_3 = -0.3$

(a) State the group of lives to whom the baseline hazard applies. [**5**]

(b) For a 72 year old patient given antibiotics 30 minutes after admission to the hospital in Los Angeles:
(i) Write down the hazard function in terms of the baseline hazard.
(ii) Find an expression for the survival function in terms of the baseline hazard. [**6**]

(c) The probability that a 44 year old man given antibiotics two hours after admission to the New York hospital is still alive a week later is 0.88. Calculate the probability that the person in (b) above will survive a week. [**10**]

**Question 3 [24 marks].** A pension scheme actuary is worried that the standard table that has been used for scheme valuations over the last ten years is now no longer suitable at the oldest ages. They compare observed mortality rates for ages 90 and above with those given by the table and compute the following standardised deviations, $z_x$ at age x:

| Age | $z_x$ | Age | $z_x$ |
|---|---|---|---|
| 90 | 0.8 | 95 | −0.3 |
| 91 | −0.4 | 96 | −1.0 |
| 92 | −1.1 | 97 | −0.6 |
| 93 | 0.6 | 98 | −0.6 |
| 94 | 2.0 | 99 | −0.8 |

(a) What type of selection is the actuary particularly concerned about here? [**2**]

(b) Complete a test of the overall goodness-of-fit of the standard table at the ages being investigated. [**10**]

(c) If the actuary thought that the table might overestimate mortality more as age increases, what further test might be particularly suitable? [**3**]

(d) Complete a standardised deviations test on this pension scheme data. [**6**]

(e) What might be the reasons for the result found in (d) above? [**3**]

 

**Question 4 [12 marks].**

(a) Explain what is meant by the **principle of correspondence** in Exposed to Risk calculations. [**2**]

(b) A coach with 20 tourists arrives at the British Museum at the precise time the museum opens where they form a line by the main entrance. The tourists have come to the museum to see an exhibition of mathematics in Ancient Greece and Rome. Each visitor has to go through security one at a time before entering the museum and this takes 2 minutes per person. Once inside, there is then a 4 minute walk to the mathematics exhibition which each tourist takes as an individual once they have cleared security. The Museum rules are that a group of ten tourists are allowed in to see the mathematics exhibits as soon as there are ten people waiting at the exhibition. These ten people are then allowed exactly 25 minutes to look at the exhibits. If there are more than ten people ready to enter, the remainder have to wait outside until the next group of ten is allowed in as soon as the exhibition room becomes free. Once they have seen the exhibition, tourists wait until everyone in their coach group has seen the exhibition before taking the same walk back to the main entrance where the coach is waiting. There is a coffee machine just outside the mathematics exhibition for people waiting either before or after their time with the exhibits. Six of the tourist group choose to have a cup of coffee from this machine. Calculate the rate of coffee consumption per person hour in the museum stating any assumptions you make. [**10**]

**Question 5 [22 marks].** An actuary in a life assurance company has used a Poisson model to estimate mortality for many years producing a single table of mortality rates by age. A recent technology upgrade at the company has introduced the possibility of the actuary using additional data on whether a policyholder is a smoker and whether they bought the policy through an independent financial advisor or through the company's distribution agreement with a supermarket chain.

(a) Would you expect mortality rates to be higher for policies bought through an advisor or policies purchased via the supermarket? Why? [**2**]

(b) List the 3 stages to building a Generalised Linear Model for mortality. [**6**]

(c) Specify fully a Generalised Linear Model for mortality at this company after the technology upgrade. [**10**]

(d) Describe how this model could be used to test your answer to (a) above. [**4**]

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**End of Paper.**