Queen Mary
University of London

Main Examination period 2022 – January – Semester A

MTH5124: Actuarial Mathematics I

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **your must submit your exam within the first 3 hours of the assessment period**.

Examiners: A. Baule, C. Sutton

Question 1 [25 marks].

(a) A saver invests £4,000 in a bank account earning interest at a nominal rate of interest of **2.5**% per annum compounded monthly. What is the balance on the account after **5** years? [**5**]

(b) Given an AER of **5**.**5**% per annum, what is the result for the corresponding nominal rate of discount compounded every **2** months? [**5**]

(c) Suppose you are paying off your new TV by monthly instalments of £**35** at the beginning of each month for the next four years. Assuming an effective interest rate of **4.5**% per annum, how much money do you have to put in a bank account to cover these payments? [**5**]

(d) Determine the present value of a perpetuity paid continuously deferred by **12** years for an AER of **5**%. [**5**]

(e) Meghan Smith is paying off a loan of £20,000 by two instalments: one payment of £10,000 one year later and one instalment of **£15,000** two years later. What is the APR charged on the loan? [**5**]

Question 2 [14 marks].
Jeanius plc is considering an investment of £10 million in a new trouser production plant. The project business plan is based on the following assumptions:

- The project will start on 1st January 2022. The initial investment is assumed to be incurred at the start of the project.
- Production will start on 1st Jannuary 2024 and run indefinitely.
- Production will be 2 million pairs of trousers each year for the first 5 years and then increased to 4 million pairs. Profits will be £0.75 per pair of trousers to be received at the end of each calendar year. Profits increase by 2% each year starting in the second year of production.
- The plant will incur production and maintenance costs of £500,000 at the beginning of each year for ten years after start of production. From the eleventh year onwards, production costs will increase by 3% each year.

Determine the Net Present Value of the project on 1st January 2022 at an effective interest rate of 5% per annum. Do not use Excel or any other software. [**14**]

Question 3 [18 marks]. A bank has issued a bond on April 1st 2020 which pays coupons at a rate of 3% of the nominal value twice a year in arrears. Coupon payments are due on 1st April and 1st October and the bond is redeemable at 110% of nominal on 1st September 2030.

(a) An investor purchases the bond on 1st March 2021. Show that the price of the bond is approximately £76 per £100 nominal assuming a gross redemption yield of 7%. [**7**]

(b) The investor pays income tax at 40%. In addition, assume that the inflation rate is constant at 1.5%. Calculate the investor's net real rate of return assuming she holds the bond until maturity. State any approximations made. [**11**]

Question 4 [21 marks].

(a) Use mortality given by table AMC00 ultimate values. You can find the table in the appendix. Determine the following quantities:

(i) The probability that a man aged 25 dies during the next 10 years. [**3**]

(ii) The probability that a man aged 67 dies between the ages of 75 and 85. [**3**]

(iii) The expected number of survivors to age 60 from a group of 2,000 men aged 25. [**4**]

(b) Consider the survival function

$$s(x) = 1 - \frac{x}{6}, \quad \text{for} \quad 0 \leq x \leq 6.$$

(i) Determine the probability density function for the complete lifetime of an individual of age 1. [**4**]

(ii) What is the probability that an individual aged 3 dies within the next 2 years? [**3**]

(iii) Calculate the complete expectation of a life at age 2 using this survival function. [**4**]

Question 5 [22 marks].

(a) Aga Khan, aged 45, takes out a whole-life assurance policy with a sum insured of £85,000 paid at the instance of death. Assume an effective interest rate of 4% per annum and mortality given by the AMC00 Select Table.

(i) Show that the cost of the life assurance policy as a net single premium is approximately £22,169. State any approximations made. [**6**]

(ii) If he pays for the assurance annually in advance until his retirement at age 67, what is the annual premium? [**4**]

(iii) If he instead pays for the assurance by making monthly payments in advance until retirement, what is the monthly premium? Use the relationship

$$\ddot{a}^{(p)}_{x:\overline{n}|} = \ddot{a}^{(p)}_{x} - {}_{n|}\ddot{a}^{(p)}_{x}$$

and state any approximations made. [**6**]

(b) Consider the function Q defined as

$$Q_x = \sum_{j=0}^{\infty} \left(l_{x+j} - l_{x+j+1}\right) v^{x+j+1},$$

where l_x is the expected number of survivors to age x. Show that in terms of Q, we can express the cost of a £1 death benefit payable at the end of the year of death as

$$A_x = \frac{Q_x}{D_x},$$

where D_x is the usual commutation function $D_x = l_x v^x$. [**6**]

End of Paper – An appendix of 3 pages follows.

AMC00 Male Life Table

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
17	9997.5094		10000.0000	17
18	9992.9305	9994.6901	9995.4200	18
19	9988.3337	9990.1025	9990.8321	19
20	9983.6991	9985.4871	9986.2163	20
21	9979.0567	9980.8438	9981.5827	21
22	9974.3666	9976.1827	9976.9213	22
23	9969.6487	9971.4740	9972.2222	23
24	9964.8832	9966.7276	9967.4854	24
25	9960.0701	9961.9336	9962.7010	25
26	9955.1996	9957.0921	9957.8691	26
27	9950.2618	9952.1832	9952.9698	27
28	9945.2469	9947.1972	9947.9933	28
29	9940.1847	9942.1340	9942.9398	29
30	9935.0655	9936.9939	9937.7794	30
31	9929.8794	9931.7671	9932.5024	31
32	9924.5767	9926.4238	9927.0892	32
33	9919.1182	9920.9344	9921.5201	33
34	9913.4841	9915.2596	9915.7755	34
35	9907.6152	9909.3799	9909.8162	35
36	9901.4922	9903.2460	9903.6126	36
37	9895.0760	9896.8385	9897.1357	37
38	9888.3275	9890.1087	9890.3363	38
39	9881.1880	9882.9977	9883.1559	39
40	9873.5893	9875.4570	9875.5558	40
41	9865.4534	9867.4085	9867.4578	41
42	9856.7223	9858.7745	9858.7942	42
43	9847.3090	9849.4875	9849.4875	43
44	9837.1169	9839.4312	9839.4312	44
45	9826.0499	9828.5291	9828.5291	45
46	9813.9735	9816.6562	9816.6562	46
47	9800.7439	9803.6786	9803.6786	47
48	9786.2090	9789.4437	9789.4437	48
49	9770.1885	9773.7708	9773.7708	49
50	9752.4550	9756.4712	9756.4712	50
51	9732.8031	9737.3192	9737.3192	51
52	9710.9228	9716.0627	9716.0627	52
53	9686.5083	9692.4138	9692.4332	53
54	9659.2100	9666.0213	9666.1182	54
55	9628.6170	9636.5205	9636.7719	55
56	9594.3554	9603.4960	9604.0069	56
57	9555.9547	9566.5317	9567.3964	57
58	9512.9284	9525.1558	9526.4767	58

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
59	9464.7096	9478.8531	9480.7305	59
60	9410.6849	9427.0212	9429.5820	60
61	9350.1805	9369.0144	9372.4010	61
62	9282.4873	9304.1589	9308.5187	62
63	9206.8320	9231.7028	9237.1968	63
64	9122.3956	9150.8360	9157.6369	64
65	9028.3257	9060.7100	9069.0001	65
66	8923.7129	8960.4327	8970.3747	66
67	8807.6361	8849.0571	8860.8106	67
68	8679.1539	8725.6105	8739.3111	68
69	8537.3365	8589.1250	8604.8568	69
70	8381.2654	8438.6022	8456.4058	70
71	8210.1056	8273.0800	8292.9181	71
72	8023.0608	8091.6666	8113.3848	72
73	7819.4936	7893.5124	7916.8461	73
74	7598.8879	7677.9138	7702.4263	74
75	7360.9680	7444.3113	7469.3894	75
76	7105.6585	7192.3651	7217.1705	76
77	6833.2053	6921.9772	6945.4296	77
78	6543.2750	6633.3614	6654.1244	78
79	6234.6920	6326.2215	6343.5631	79
80	5905.8869	6000.3050	6014.4463	80
81	5557.1270	5655.4241	5667.9541	81
82	5191.0860	5293.5691	5305.7888	82
83	4811.5773	4917.5469	4930.2080	83
84	4422.4381	4531.0383	4544.0494	84
85	4027.2597	4137.4915	4150.7347	85
86	3630.1730	3740.8772	3754.2191	86
87	3235.7358	3345.6473	3358.9374	87
88	2848.8406	2956.5986	2969.6769	88
89	2474.5254	2578.7306	2591.4291	89
90	2117.7709	2217.0436	2229.1940	90
91		1876.3069	1887.7528	91
92			1571.4202	92
93			1283.8047	93
94			1027.5920	94
95			804.3661	95
96			614.5164	96
97			457.2229	97
98			330.5502	98
99			231.6268	99
100			156.9026	100

AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17
18	4932.79	4933.66	4934.02	18
19	4740.89	4741.73	4742.07	19
20	4556.43	4557.25	4557.58	20
21	4379.15	4379.93	4380.25	21
22	4208.74	4209.50	4209.82	22
23	4044.95	4045.69	4045.99	23
24	3887.51	3888.23	3888.53	24
25	3736.19	3736.89	3737.18	25
26	3590.73	3591.42	3591.70	26
27	3450.92	3451.58	3451.85	27
28	3316.52	3317.17	3317.43	28
29	3187.33	3187.96	3188.22	29
30	3063.17	3063.76	3064.00	30
31	2943.81	2944.37	2944.59	31
32	2829.08	2829.61	2829.80	32
33	2718.77	2719.27	2719.43	33
34	2612.72	2613.19	2613.32	34
35	2510.74	2511.19	2511.30	35
36	2412.68	2413.11	2413.20	36
37	2318.39	2318.80	2318.87	37
38	2227.70	2228.10	2228.15	38
39	2140.47	2140.86	2140.90	39
40	2056.56	2056.95	2056.97	40
41	1975.83	1976.22	1976.23	41
42	1898.16	1898.55	1898.56	42
43	1823.41	1823.81	1823.81	43
44	1751.46	1751.87	1751.87	44
45	1682.20	1682.63	1682.63	45
46	1615.52	1615.96	1615.96	46
47	1551.29	1551.75	1551.75	47
48	1489.41	1489.90	1489.90	48
49	1429.78	1430.30	1430.30	49
50	1372.29	1372.86	1372.86	50
51	1316.85	1317.47	1317.47	51
52	1263.36	1264.03	1264.03	52
53	1211.71	1212.45	1212.46	53
54	1161.83	1162.65	1162.66	54
55	1113.60	1114.52	1114.55	55
56	1066.96	1067.98	1068.03	56
57	1021.82	1022.95	1023.04	57
58	978.09	979.35	979.49	58

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
59	935.71	937.11	937.29	59
60	894.58	896.14	896.38	60
61	854.65	856.37	856.68	61
62	815.83	817.73	818.11	62
63	778.05	780.16	780.62	63
64	741.27	743.58	744.13	64
65	705.41	707.94	708.59	65
66	670.42	673.18	673.92	66
67	636.25	639.24	640.09	67
68	602.85	606.08	607.03	68
69	570.19	573.65	574.70	69
70	538.24	541.92	543.07	70
71	506.97	510.86	512.08	71
72	476.36	480.44	481.73	72
73	446.42	450.65	451.98	73
74	417.14	421.48	422.82	74
75	388.54	392.94	394.26	75
76	360.64	365.04	366.30	76
77	333.47	337.80	338.95	77
78	307.04	311.27	312.24	78
79	281.31	285.44	286.22	79
80	256.22	260.32	260.93	80
81	231.82	235.92	236.44	81
82	208.22	212.33	212.82	82
83	185.58	189.66	190.15	83
84	164.01	168.03	168.52	84
85	143.61	147.54	148.01	85
86	124.47	128.26	128.72	86
87	106.68	110.30	110.74	87
88	90.31	93.73	94.14	88
89	75.43	78.60	78.99	89
90	62.07	64.98	65.34	90
91		52.88	53.20	91
92			42.58	92
93			33.45	93
94			25.74	94
95			19.38	95
96			14.23	96
97			10.18	97
98			7.08	98
99			4.77	99
100			3.11	100

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AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17
18	115879.55	115880.77	115881.13	18
19	110945.59	110946.76	110947.11	19
20	106203.56	106204.70	106205.03	20
21	101646.03	101647.13	101647.45	21
22	97265.82	97266.89	97267.20	22
23	93056.05	93057.08	93057.39	23
24	89010.09	89011.10	89011.39	24
25	85121.59	85122.57	85122.86	25
26	81384.45	81385.41	81385.69	26
27	77792.78	77793.72	77793.99	27
28	74340.96	74341.87	74342.13	28
29	71023.58	71024.44	71024.70	29
30	67835.43	67836.24	67836.49	30
31	64771.51	64772.26	64772.48	31
32	61827.01	61827.70	61827.89	32
33	58997.30	58997.93	58998.09	33
34	56277.95	56278.53	56278.66	34
35	53664.69	53665.23	53665.34	35
36	51153.45	51153.95	51154.04	36
37	48740.30	48740.77	48740.84	37
38	46421.48	46421.92	46421.97	38
39	44193.38	44193.79	44193.82	39
40	42052.51	42052.91	42052.93	40
41	39995.55	39995.95	39995.96	41
42	38019.32	38019.72	38019.72	42
43	36120.76	36121.17	36121.17	43
44	34296.94	34297.35	34297.35	44
45	32545.06	32545.48	32545.48	45
46	30862.41	30862.85	30862.85	46
47	29246.43	29246.89	29246.89	47
48	27694.65	27695.14	27695.14	48
49	26204.72	26205.24	26205.24	49
50	24774.37	24774.94	24774.94	50
51	23401.47	23402.08	23402.08	51
52	22083.94	22084.61	22084.61	52
53	20819.83	20820.58	20820.58	53
54	19607.27	19608.12	19608.13	54
55	18444.47	18445.44	18445.47	55
56	17329.76	17330.87	17330.93	56
57	16261.53	16262.80	16262.89	57
58	15238.27	15239.71	15239.85	58

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
59	14258.53	14260.18	14260.36	59
60	13320.96	13322.83	13323.07	60
61	12424.28	12426.38	12426.69	61
62	11567.26	11569.63	11570.01	62
63	10748.78	10751.44	10751.90	63
64	9967.77	9970.73	9971.28	64
65	9223.22	9226.50	9227.15	65
66	8514.21	8517.82	8518.56	66
67	7839.85	7843.79	7844.64	67
68	7199.32	7203.60	7204.55	68
69	6591.87	6596.47	6597.52	69
70	6016.77	6021.68	6022.82	70
71	5473.35	5478.53	5479.75	71
72	4960.98	4966.38	4967.67	72
73	4479.04	4484.61	4485.94	73
74	4026.96	4032.62	4033.96	74
75	3604.16	3609.81	3611.14	75
76	3210.07	3215.62	3216.88	76
77	2844.13	2849.44	2850.58	77
78	2505.65	2510.66	2511.63	78
79	2193.87	2198.61	2199.39	79
80	1907.94	1912.56	1913.17	80
81	1647.13	1651.72	1652.24	81
82	1410.71	1415.31	1415.80	82
83	1197.92	1202.49	1202.97	83
84	1007.84	1012.34	1012.82	84
85	839.45	843.83	844.31	85
86	691.61	695.84	696.30	86
87	563.10	567.14	567.58	87
88	452.62	456.42	456.84	88
89	358.78	362.31	362.70	89
90	280.12	283.35	283.71	90
91		218.05	218.37	91
92			165.17	92
93			122.59	93
94			89.14	94
95			63.39	95
96			44.02	96
97			29.78	97
98			19.60	98
99			12.52	99
100			7.75	100

End of Appendix.