Queen Mary
University of London

Main Examination period 2020 – January – Semester A

MTH5124: Actuarial Mathematics I

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Examiners: A. Baule, C. Sutton

Question 1 [22 marks].

(a) A saver invests £2,000 in a bank account earning interest at an AER of 2.5%. What is the balance on the account after 5 years? **[4]**

(b) A bank pays interest at a nominal interest rate of 3% per annum compounded monthly. What is the corresponding effective interest rate over 6 months? **[4]**

(c) An individual wins a lottery prize of £5,000 payable every month in advance for 20 years. What is the present value of the cashflow at a nominal interest rate of 4% per annum compounded monthly? **[6]**

(d) Suppose that you take out a £100 loan from a money lender with interest compounded daily. You do not make any repayments on the loan for a full year and at the end of the year you owe £2500. What is the nominal annual interest rate for the loan? **[4]**

(e) Explain in words the meaning of the symbol $\bar{s}_{\overline{15}|}$ at an effective rate of interest of $i\%$ per annum. **[4]**

Question 2 [18 marks]. Ms Smith is paying off a loan of £10,000 taken out on June 1st 2016 over ten years. Repayments are paid monthly in arrears and the AER charged is 15%.

(a) Show that the monthly repayment amount is approximately £156. **[5]**

(b) Show that the amount of the outstanding debt on June 1st 2019 is approximately £8290. **[5]**

(c) A second bank has agreed to lend Ms Smith the amount to pay off this outstanding debt on June 1st 2019 over the remaining time period of the original loan. The AER of the new loan is 10%. Repayments of £180 are to be made monthly in advance. On what date will she make the final repayment? **[8]**

Question 3 [16 marks]. On 1 July 2017, an investor buys a corporate bond issued by BigCo plc. The bond is repayable at par on 1 July 2037 and has an annual coupon rate of 3% payable twice yearly in arrears.

(a) Assuming that the gross redemption yield on the bond is 8.0% per annum calculate the purchase price per £100 nominal. **[6]**

(b) BigCo plc has some financial problems and it looks as if the company may not be able to make all repayments on the bond. The price of the bond falls, and on 1 July 2019 the gross redemption yield on the bond is 11% per annum. On the same day, a bank makes an offer to buy the company if bond holders accept an immediate payment of £54 per £100 nominal. Calculate the price of the bond on 1 July 2019 and explain whether or not the investor should accept this offer. **[7]**

(c) Assuming that the investor pays income tax at the rate of 40% how much tax has the investor paid on the bond investment between 1 July 2017 and 1 July 2019? **[3]**

Question 4 [18 marks]. Consider the survival function

$$s(x) = 1 - \frac{x}{50}, \qquad \text{for} \quad 0 \leqslant x \leqslant 50.$$

(a) Show that $s(x)$ is a valid survival function. **[6]**

(b) Determine the expected number of survivors to age 30 from a group of 500 20-year olds using $s(x)$. **[4]**

(c) Derive an expression for the force of mortality μ_x using $s(x)$. **[4]**

(d) Calculate the complete expectation of life at age $x = 5$ using $s(x)$. **[4]**

Question 5 [14 marks]. Mr Raza plans to retire at age 65 with a pension of £30,000 per year, paid annually in advance during his remaining lifetime, payable from 65. Assume an effective interest rate of 4% per annum and mortality given by the AMC00 Select Table. You can find the table in the Appendix.

(a) Show that the expected present value of the pension at retirement is approximately £392,249. **[6]**

(b) In order to cover the cost of the pension, Mr Raza wants to make monthly savings in arrears starting at age 40 until retirement. How much does he have to save each month? Assume that he survives until retirement. **[6]**

(c) How does the expected present value in (a) change when he switches to a pension of £2,500 paid monthly in advance during his remaining lifetime? Give reasons. **[2]**

Question 6 [12 marks]. Dr Lin, aged 35, takes out a pure endowment policy, which will pay a survival benefit of £60,000 if he survives to age 55. Assume an effective interest rate of 4% per annum and mortality given by the AMC00 Select Table.

(a) What is the cost of the endowment policy? **[6]**

(b) Dr Lin plans to use the survival benefit to purchase a whole-life annuity paying a monthly amount in advance. How much are the monthly payments? **[6]**

End of Paper – An appendix of 5 pages follows.

AMC00 Male Life Table

Age x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	Age x
17	9997.5094		10000.0000	17
18	9992.9305	9994.6901	9995.4200	18
19	9988.3337	9990.1025	9990.8321	19
20	9983.6991	9985.4871	9986.2163	20
21	9979.0567	9980.8438	9981.5827	21
22	9974.3666	9976.1827	9976.9213	22
23	9969.6487	9971.4740	9972.2222	23
24	9964.8832	9966.7276	9967.4854	24
25	9960.0701	9961.9336	9962.7010	25
26	9955.1996	9957.0921	9957.8691	26
27	9950.2618	9952.1832	9952.9698	27
28	9945.2469	9947.1972	9947.9933	28
29	9940.1847	9942.1340	9942.9398	29
30	9935.0655	9936.9939	9937.7794	30
31	9929.8794	9931.7671	9932.5024	31
32	9924.5767	9926.4238	9927.0892	32
33	9919.1182	9920.9344	9921.5201	33
34	9913.4841	9915.2596	9915.7755	34
35	9907.6152	9909.3799	9909.8162	35
36	9901.4922	9903.2460	9903.6126	36
37	9895.0760	9896.8385	9897.1357	37
38	9888.3275	9890.1087	9890.3363	38
39	9881.1880	9882.9977	9883.1559	39
40	9873.5893	9875.4570	9875.5558	40
41	9865.4534	9867.4085	9867.4578	41
42	9856.7223	9858.7745	9858.7942	42
43	9847.3090	9849.4875	9849.4875	43
44	9837.1169	9839.4312	9839.4312	44
45	9826.0499	9828.5291	9828.5291	45
46	9813.9735	9816.6562	9816.6562	46
47	9800.7439	9803.6786	9803.6786	47
48	9786.2090	9789.4437	9789.4437	48
49	9770.1885	9773.7708	9773.7708	49
50	9752.4550	9756.4712	9756.4712	50
51	9732.8031	9737.3192	9737.3192	51
52	9710.9228	9716.0627	9716.0627	52
53	9686.5083	9692.4138	9692.4332	53
54	9659.2100	9666.0213	9666.1182	54
55	9628.6170	9636.5205	9636.7719	55
56	9594.3554	9603.4960	9604.0069	56
57	9555.9547	9566.5317	9567.3964	57
58	9512.9284	9525.1558	9526.4767	58

Age x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	Age x
59	9464.7096	9478.8531	9480.7305	59
60	9410.6849	9427.0212	9429.5820	60
61	9350.1805	9369.0144	9372.4010	61
62	9282.4873	9304.1589	9308.5187	62
63	9206.8320	9231.7028	9237.1968	63
64	9122.3956	9150.8360	9157.6369	64
65	9028.3257	9060.7100	9069.0001	65
66	8923.7129	8960.4327	8970.3747	66
67	8807.6361	8849.0571	8860.8106	67
68	8679.1539	8725.6105	8739.3111	68
69	8537.3365	8589.1250	8604.8568	69
70	8381.2654	8438.6022	8456.4058	70
71	8210.1056	8273.0800	8292.9181	71
72	8023.0608	8091.6666	8113.3848	72
73	7819.4936	7893.5124	7916.8461	73
74	7598.8879	7677.9138	7702.4263	74
75	7360.9680	7444.3113	7469.3894	75
76	7105.6585	7192.3651	7217.1705	76
77	6833.2053	6921.9772	6945.4296	77
78	6543.2750	6633.3614	6654.1244	78
79	6234.6920	6326.2215	6343.5631	79
80	5905.8869	6000.3050	6014.4463	80
81	5557.1270	5655.4241	5667.9541	81
82	5191.0860	5293.5691	5305.7888	82
83	4811.5773	4917.5469	4930.2080	83
84	4422.4381	4531.0383	4544.0494	84
85	4027.2597	4137.4915	4150.7347	85
86	3630.1730	3740.8772	3754.2191	86
87	3235.7358	3345.6473	3358.9374	87
88	2848.8406	2956.5986	2969.6769	88
89	2474.5254	2578.7306	2591.4291	89
90	2117.7709	2217.0436	2229.1940	90
91		1876.3069	1887.7528	91
92			1571.4202	92
93			1283.8047	93
94			1027.5920	94
95			804.3661	95
96			614.5164	96
97			457.2229	97
98			330.5502	98
99			231.6268	99
100			156.9026	100

AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17
18	4932.79	4933.66	4934.02	18
19	4740.89	4741.73	4742.07	19
20	4556.43	4557.25	4557.58	20
21	4379.15	4379.93	4380.25	21
22	4208.74	4209.50	4209.82	22
23	4044.95	4045.69	4045.99	23
24	3887.51	3888.23	3888.53	24
25	3736.19	3736.89	3737.18	25
26	3590.73	3591.42	3591.70	26
27	3450.92	3451.58	3451.85	27
28	3316.52	3317.17	3317.43	28
29	3187.33	3187.96	3188.22	29
30	3063.17	3063.76	3064.00	30
31	2943.81	2944.37	2944.59	31
32	2829.08	2829.61	2829.80	32
33	2718.77	2719.27	2719.43	33
34	2612.72	2613.19	2613.32	34
35	2510.74	2511.19	2511.30	35
36	2412.68	2413.11	2413.20	36
37	2318.39	2318.80	2318.87	37
38	2227.70	2228.10	2228.15	38
39	2140.47	2140.86	2140.90	39
40	2056.56	2056.95	2056.97	40
41	1975.83	1976.22	1976.23	41
42	1898.16	1898.55	1898.56	42
43	1823.41	1823.81	1823.81	43
44	1751.46	1751.87	1751.87	44
45	1682.20	1682.63	1682.63	45
46	1615.52	1615.96	1615.96	46
47	1551.29	1551.75	1551.75	47
48	1489.41	1489.90	1489.90	48
49	1429.78	1430.30	1430.30	49
50	1372.29	1372.86	1372.86	50
51	1316.85	1317.47	1317.47	51
52	1263.36	1264.03	1264.03	52
53	1211.71	1212.45	1212.46	53
54	1161.83	1162.65	1162.66	54
55	1113.60	1114.52	1114.55	55
56	1066.96	1067.98	1068.03	56
57	1021.82	1022.95	1023.04	57
58	978.09	979.35	979.49	58

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
59	935.71	937.11	937.29	59
60	894.58	896.14	896.38	60
61	854.65	856.37	856.68	61
62	815.83	817.73	818.11	62
63	778.05	780.16	780.62	63
64	741.27	743.58	744.13	64
65	705.41	707.94	708.59	65
66	670.42	673.18	673.92	66
67	636.25	639.24	640.09	67
68	602.85	606.08	607.03	68
69	570.19	573.65	574.70	69
70	538.24	541.92	543.07	70
71	506.97	510.86	512.08	71
72	476.36	480.44	481.73	72
73	446.42	450.65	451.98	73
74	417.14	421.48	422.82	74
75	388.54	392.94	394.26	75
76	360.64	365.04	366.30	76
77	333.47	337.80	338.95	77
78	307.04	311.27	312.24	78
79	281.31	285.44	286.22	79
80	256.22	260.32	260.93	80
81	231.82	235.92	236.44	81
82	208.22	212.33	212.82	82
83	185.58	189.66	190.15	83
84	164.01	168.03	168.52	84
85	143.61	147.54	148.01	85
86	124.47	128.26	128.72	86
87	106.68	110.30	110.74	87
88	90.31	93.73	94.14	88
89	75.43	78.60	78.99	89
90	62.07	64.98	65.34	90
91		52.88	53.20	91
92			42.58	92
93			33.45	93
94			25.74	94
95			19.38	95
96			14.23	96
97			10.18	97
98			7.08	98
99			4.77	99
100			3.11	100

AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17
18	115879.55	115880.77	115881.13	18
19	110945.59	110946.76	110947.11	19
20	106203.56	106204.70	106205.03	20
21	101646.03	101647.13	101647.45	21
22	97265.82	97266.89	97267.20	22
23	93056.05	93057.08	93057.39	23
24	89010.09	89011.10	89011.39	24
25	85121.59	85122.57	85122.86	25
26	81384.45	81385.41	81385.69	26
27	77792.78	77793.72	77793.99	27
28	74340.96	74341.87	74342.13	28
29	71023.58	71024.44	71024.70	29
30	67835.43	67836.24	67836.49	30
31	64771.51	64772.26	64772.48	31
32	61827.01	61827.70	61827.89	32
33	58997.30	58997.93	58998.09	33
34	56277.95	56278.53	56278.66	34
35	53664.69	53665.23	53665.34	35
36	51153.45	51153.95	51154.04	36
37	48740.30	48740.77	48740.84	37
38	46421.48	46421.92	46421.97	38
39	44193.38	44193.79	44193.82	39
40	42052.51	42052.91	42052.93	40
41	39995.55	39995.95	39995.96	41
42	38019.32	38019.72	38019.72	42
43	36120.76	36121.17	36121.17	43
44	34296.94	34297.35	34297.35	44
45	32545.06	32545.48	32545.48	45
46	30862.41	30862.85	30862.85	46
47	29246.43	29246.89	29246.89	47
48	27694.65	27695.14	27695.14	48
49	26204.72	26205.24	26205.24	49
50	24774.37	24774.94	24774.94	50
51	23401.47	23402.08	23402.08	51
52	22083.94	22084.61	22084.61	52
53	20819.83	20820.58	20820.58	53
54	19607.27	19608.12	19608.13	54
55	18444.47	18445.44	18445.47	55
56	17329.76	17330.87	17330.93	56
57	16261.53	16262.80	16262.89	57
58	15238.27	15239.71	15239.85	58

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
59	14258.53	14260.18	14260.36	59
60	13320.96	13322.83	13323.07	60
61	12424.28	12426.38	12426.69	61
62	11567.26	11569.63	11570.01	62
63	10748.78	10751.44	10751.90	63
64	9967.77	9970.73	9971.28	64
65	9223.22	9226.50	9227.15	65
66	8514.21	8517.82	8518.56	66
67	7839.85	7843.79	7844.64	67
68	7199.32	7203.60	7204.55	68
69	6591.87	6596.47	6597.52	69
70	6016.77	6021.68	6022.82	70
71	5473.35	5478.53	5479.75	71
72	4960.98	4966.38	4967.67	72
73	4479.04	4484.61	4485.94	73
74	4026.96	4032.62	4033.96	74
75	3604.16	3609.81	3611.14	75
76	3210.07	3215.62	3216.88	76
77	2844.13	2849.44	2850.58	77
78	2505.65	2510.66	2511.63	78
79	2193.87	2198.61	2199.39	79
80	1907.94	1912.56	1913.17	80
81	1647.13	1651.72	1652.24	81
82	1410.71	1415.31	1415.80	82
83	1197.92	1202.49	1202.97	83
84	1007.84	1012.34	1012.82	84
85	839.45	843.83	844.31	85
86	691.61	695.84	696.30	86
87	563.10	567.14	567.58	87
88	452.62	456.42	456.84	88
89	358.78	362.31	362.70	89
90	280.12	283.35	283.71	90
91		218.05	218.37	91
92			165.17	92
93			122.59	93
94			89.14	94
95			63.39	95
96			44.02	96
97			29.78	97
98			19.60	98
99			12.52	99
100			7.75	100

AMC00 Male Life Functions (4%)

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
17	478.10		479.31	17
18	475.89	476.71	477.05	18
19	473.75	474.54	474.88	19
20	471.68	472.45	472.77	20
21	469.68	470.42	470.74	21
22	467.74	468.47	468.77	22
23	465.87	466.57	466.86	23
24	464.05	464.73	465.02	24
25	462.28	462.94	463.22	25
26	460.56	461.21	461.48	26
27	458.89	459.52	459.78	27
28	457.25	457.86	458.12	28
29	455.66	456.25	456.50	29
30	454.11	454.67	454.91	30
31	452.60	453.13	453.34	31
32	451.12	451.62	451.80	32
33	449.65	450.12	450.27	33
34	448.18	448.63	448.76	34
35	446.72	447.14	447.25	35
36	445.24	445.65	445.74	36
37	443.76	444.15	444.22	37
38	442.25	442.64	442.69	38
39	440.72	441.10	441.13	39
40	439.16	439.53	439.55	40
41	437.54	437.92	437.93	41
42	435.88	436.26	436.26	42
43	434.15	434.54	434.54	43
44	432.35	432.75	432.75	44
45	430.47	430.88	430.88	45
46	428.50	428.92	428.92	46
47	426.42	426.87	426.87	47
48	424.23	424.70	424.70	48
49	421.91	422.41	422.41	49
50	419.43	419.98	419.98	50
51	416.80	417.39	417.39	51
52	413.98	414.62	414.62	52
53	410.95	411.66	411.66	53
54	407.70	408.49	408.50	54
55	404.20	405.08	405.10	55
56	400.43	401.41	401.46	56
57	396.37	397.46	397.55	57
58	392.01	393.21	393.34	58

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
59	387.30	388.64	388.82	59
60	382.24	383.72	383.95	60
61	376.79	378.43	378.73	61
62	370.93	372.74	373.11	62
63	364.64	366.64	367.09	63
64	357.89	360.09	360.62	64
65	350.67	353.07	353.70	65
66	342.95	345.57	346.29	66
67	334.71	337.55	338.37	67
68	325.95	329.02	329.93	68
69	316.66	319.94	320.95	69
70	306.83	310.32	311.42	70
71	296.46	300.15	301.32	71
72	285.56	289.42	290.66	72
73	274.15	278.16	279.44	73
74	262.26	266.38	267.67	74
75	249.92	254.10	255.37	75
76	237.17	241.36	242.57	76
77	224.08	228.21	229.31	77
78	210.67	214.70	215.64	78
79	196.93	200.88	201.63	79
80	182.84	186.76	187.35	80
81	168.47	172.39	172.90	81
82	153.96	157.90	158.37	82
83	139.50	143.41	143.88	83
84	125.24	129.10	129.56	84
85	111.32	115.08	115.54	85
86	97.87	101.50	101.94	86
87	85.02	88.49	88.91	87
88	72.90	76.17	76.57	88
89	61.63	64.67	65.04	89
90	51.30	54.08	54.42	90
91		44.49	44.80	91
92			36.23	92
93			28.74	93
94			22.32	94
95			16.94	95
96			12.54	96
97			9.04	97
98			6.33	98
99			4.29	99
100			2.81	100

AMC00 Male Life Functions (4%)

Age x	$A_{[x]}$	A_x	$\ddot{a}_{[x]}$	$\ddot{a}_x$	Age x
17	0.09315	0.09337	23.578	23.572	17
18	0.09647	0.09669	23.492	23.486	18
19	0.09993	0.10014	23.402	23.396	19
20	0.10352	0.10373	23.309	23.303	20
21	0.10725	0.10747	23.211	23.206	21
22	0.11114	0.11135	23.110	23.105	22
23	0.11517	0.11539	23.005	23.000	23
24	0.11937	0.11959	22.896	22.891	24
25	0.12373	0.12395	22.783	22.777	25
26	0.12826	0.12848	22.665	22.659	26
27	0.13297	0.13320	22.543	22.537	27
28	0.13787	0.13809	22.415	22.410	28
29	0.14296	0.14318	22.283	22.277	29
30	0.14825	0.14847	22.146	22.140	30
31	0.15375	0.15396	22.003	21.997	31
32	0.15946	0.15966	21.854	21.849	32
33	0.16539	0.16558	21.700	21.695	33
34	0.17154	0.17172	21.540	21.535	34
35	0.17792	0.17809	21.374	21.370	35
36	0.18454	0.18471	21.202	21.198	36
37	0.19141	0.19157	21.023	21.019	37
38	0.19853	0.19868	20.838	20.834	38
39	0.20590	0.20605	20.647	20.643	39
40	0.21354	0.21369	20.448	20.444	40
41	0.22145	0.22160	20.242	20.238	41
42	0.22963	0.22978	20.030	20.026	42
43	0.23810	0.23826	19.809	19.805	43
44	0.24685	0.24702	19.582	19.578	44
45	0.25590	0.25608	19.347	19.342	45
46	0.26524	0.26543	19.104	19.099	46
47	0.27488	0.27509	18.853	18.848	47
48	0.28483	0.28506	18.594	18.589	48
49	0.29508	0.29533	18.328	18.321	49
50	0.30564	0.30591	18.053	18.046	50
51	0.31651	0.31681	17.771	17.763	51
52	0.32768	0.32801	17.480	17.472	52
53	0.33915	0.33953	17.182	17.172	53
54	0.35091	0.35135	16.876	16.865	54
55	0.36297	0.36347	16.563	16.550	55
56	0.37530	0.37589	16.242	16.227	56
57	0.38791	0.38859	15.914	15.897	57
58	0.40079	0.40158	15.580	15.559	58

Age x	$A_{[x]}$	A_x	$\ddot{a}_{[x]}$	$\ddot{a}_x$	Age x
59	0.41391	0.41483	15.238	15.214	59
60	0.42728	0.42834	14.891	14.863	60
61	0.44087	0.44209	14.537	14.506	61
62	0.45467	0.45607	14.179	14.142	62
63	0.46866	0.47025	13.815	13.774	63
64	0.48281	0.48462	13.447	13.400	64
65	0.49711	0.49916	13.075	13.022	65
66	0.51154	0.51384	12.700	12.640	66
67	0.52608	0.52863	12.322	12.256	67
68	0.54069	0.54352	11.942	11.869	68
69	0.55536	0.55847	11.561	11.480	69
70	0.57005	0.57345	11.179	11.090	70
71	0.58476	0.58843	10.796	10.701	71
72	0.59945	0.60338	10.414	10.312	72
73	0.61411	0.61827	10.033	9.925	73
74	0.62870	0.63306	9.654	9.541	74
75	0.64322	0.64772	9.276	9.159	75
76	0.65765	0.66222	8.901	8.782	76
77	0.67197	0.67653	8.529	8.410	77
78	0.68613	0.69062	8.161	8.044	78
79	0.70005	0.70445	7.799	7.684	79
80	0.71360	0.71800	7.446	7.332	80
81	0.72672	0.73123	7.105	6.988	81
82	0.73942	0.74413	6.775	6.652	82
83	0.75173	0.75668	6.455	6.326	83
84	0.76365	0.76884	6.145	6.010	84
85	0.77518	0.78060	5.845	5.704	85
86	0.78629	0.79195	5.556	5.409	86
87	0.79698	0.80287	5.279	5.125	87
88	0.80724	0.81336	5.012	4.853	88
89	0.81705	0.82340	4.757	4.592	89
90	0.82642	0.83299	4.513	4.342	90
91		0.84213		4.105	91
92		0.85081		3.879	92
93		0.85905		3.665	93
94		0.86683		3.462	94
95		0.87417		3.272	95
96		0.88106		3.092	96
97		0.88751		2.925	97
98		0.89351		2.769	98
99		0.89904		2.625	99
100		0.90404		2.495	100

End of Appendix.