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9. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi3.pg`
Are the following statements true or false for a square matrix A?

? 1. If **u** and **v** are linearly independent eigenvectors, then they correspond to distinct eigenvalues.
? 2. Finding an eigenvector of A might be difficult, but checking whether a given vector is in fact an eigenvector is easy.
? 3. A matrix A is singular if and only if 0 is an eigenvalue of A.
? 4. An $n \times n$ matrix A is diagonalizable if A has n linearly independent eigenvectors.
? 5. The eigenvalues of a matrix are the entries on its main diagonal.

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8. **(8 points)** `local/Library/WHFreeman/Holt_linear_algebra/Chaps_1-4/4.3.6.pg`
Find bases for the column space, the row space, and the nullspace of the matrix

$$A = \begin{bmatrix} 1 & 5 & -1 & 1 \\ 2 & 13 & 0 & 3 \\ 2 & 16 & 2 & 4 \end{bmatrix}$$

Basis for the column space of A = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

Basis for the row space of A = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

Basis for the null space of A = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

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11. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi4.pg`
Are the following statements true or false?

? 1. For all vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, we have $\mathbf{u} \cdot \mathbf{v} = -\mathbf{v} \cdot \mathbf{u}$.
? 2. For a square matrix A, vectors in the column space of A are orthogonal to vectors in the nullspace of A.
? 3. For any scalar c and any vector $\mathbf{v} \in \mathbb{R}^n$, $||c\mathbf{v}|| = c||\mathbf{v}||$.
? 4. If an $n \times p$ matrix U has orthonormal columns, then $UU^T\mathbf{x} = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$.
? 5. If vectors $\mathbf{v}_1, ..., \mathbf{v}_p$ span a subspace W and if $\mathbf{x}$ is orthogonal to each $\mathbf{v}_j$ for $j = 1, ..., p$, then $\mathbf{x}$ is in $W^{\perp}$.

1. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi1.pg`
Are the following statements true or false?

? 1. If A is a singular matrix, the system $A\mathbf{x} = \mathbf{0}$ is inconsistent.
? 2. If A is an invertible upper triangular matrix, then A^{-1} is lower triangular.
? 3. If A is a square matrix satisfying $A^3 = I$, then A is invertible.
? 4. The linear system $A\mathbf{x} = \mathbf{b}$ will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as the columns of the matrix A do not include the zero column.
? 5. If A is a 2×2 matrix, then $\det(\operatorname{adj}(A)) = \det(A)$

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12. **(7 points)** `local/Library/Rochester/setLinearAlgebra20LeastSquares/ur_la_20_5.pg`
Fit a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(-4, -3)$, $(0, 2)$, $(4, 13)$, using the least squares method.

$f(t) =$ ____________

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6. **(5 points)** `local/Library/Hope/Multi1/04-04-Basis-and-functions/Matrix_rep_03.pg`
Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$L(\mathbf{x}) = \begin{bmatrix} 0 & 4 & 0 \\ 5 & -4 & 0 \\ 2 & -1 & -5 \end{bmatrix} \mathbf{x}.$$

Let

$$\begin{aligned} \mathcal{B} &= \{\langle 0, 1, -1 \rangle, \langle 0, 2, -1 \rangle, \langle 1, 2, -1 \rangle\}, \\ \mathcal{C} &= \{\langle 1, -1, -1 \rangle, \langle 1, -2, -1 \rangle, \langle 3, -3, -2 \rangle\}, \end{aligned}$$

be two different bases for $\mathbb{R}^3$. Find the matrix $[L]_{\mathcal{C}}^{\mathcal{B}}$ for L relative to the basis $\mathcal{B}$ in the domain and $\mathcal{C}$ in the codomain.

$$[L]_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} \_\_\_\_ & \_\_ & \_\_ \\ \_\_\_\_ & \_\_ & \_\_ \\ \_\_\_\_ & \_\_ & \_\_ \end{bmatrix}$$

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10. **(8 points)** `local/Library/Rochester/setLinearAlgebra12Diagonalization/ur_la_12_12.pg`
Let

$$M = \begin{bmatrix} 5 & -1 \\ 2 & 2 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

Hint: a formula such as $5 \cdot (2.3)^n + 7 \cdot (3.5)^n$ is typeset as $5*(2.3)^n + 7*(3.5)^n$.

$$M^n = \begin{bmatrix} \_\_\_\_\_\_\_\_\_\_ & \_\_ \\ \_\_\_\_\_\_ & \_\_ \end{bmatrix}$$

7. **(7 points)** local/Library/Rochester/setLinearAlgebra15TransfOfLinSpaces/ur_la_15_13.pg

Let V be the subspace of the vector space of continuous functions on $\mathbb{R}$ spanned by the functions $\cos(t)$ and $\sin(t)$.

Consider the linear transformation $T : V \to V$ given by

$$(T(f))(t) = f''(t) + 9f'(t) + 5f(t),$$

for $f \in V$.

Find the matrix A associated to T with respect to the basis $(\cos(t), \sin(t))$.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

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3. **(10 points)** local/setSemester_A_final_assessment_2021-22/span.pg

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be (non-zero) vectors and suppose that $\mathbf{z} = 1\mathbf{x} + 1\mathbf{y}$ and $\mathbf{w} = 5\mathbf{x} + 5\mathbf{y} - 4\mathbf{z}$.

Are the following statements true or false?

[?] 1. Span($\mathbf{x}, \mathbf{y}$) = Span($\mathbf{w}, \mathbf{x}, \mathbf{z}$)

[?] 2. Span($\mathbf{w}, \mathbf{x}, \mathbf{z}$) = Span($\mathbf{w}, \mathbf{z}$)

[?] 3. Span($\mathbf{x}, \mathbf{z}$) = Span($\mathbf{w}, \mathbf{y}$)

[?] 4. Span($\mathbf{w}, \mathbf{z}$) = Span($\mathbf{x}, \mathbf{z}$)

[?] 5. Span($\mathbf{w}, \mathbf{x}$) = Span($\mathbf{y}, \mathbf{z}$)

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5. **(10 points)** local/setSemester_A_final_assessment_2021-22/new1Problem.pg

Determine whether the given set S is a subspace of the vector space V.

[?] 1. $V = \mathbb{R}^{n \times n}$, and S is the subset of all matrices A satisfying $A^T = -A$.

[?] 2. $V = C^2(\mathbb{R})$, and S is the subset of V consisting of those functions satisfying the differential equation $y'' - 4y' + 3y = 0$.

[?] 3. $V = \mathbb{R}^{n \times n}$, and S is the subset of all $n \times n$ matrices A with $\det(A) = 0$.

[?] 4. $V = P_n$, and S is the subset of P_n consisting of those polynomials satisfying $p(0) = 0$.

[?] 5. $V = \mathbb{R}^{n \times n}$, and S is the subset of all nonsingular matrices.

Notation: P_n is the vector space of polynomials of degree up to n, and $C^n(\mathbb{R})$ is the vector space of n times continuously differentiable functions on $\mathbb{R}$.

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4. **(5 points)** Library/Rochester/setLinearAlgebra9Dependence/ur_la_9_7.pg

The vectors

$$\vec{u} = \begin{bmatrix} -4 \\ 11 \\ 30 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 4 \\ -8 \\ -17+k \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

are linearly independent if and only if $k \neq$ _____.

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2. **(10 points)** local/setSemester_A_final_assessment_2021-22/multi2.pg

Are the following statements true or false?

[?] 1. The space P_n of polynomials of degree up to n has a basis consisting of polynomials that all have the same degree.

[?] 2. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set, then $\{\mathbf{u} + 4\mathbf{v}, \mathbf{v} - 7\mathbf{w}, \mathbf{w}\}$ is linearly independent.

[?] 3. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set, then $\{2\mathbf{u} + 4\mathbf{v} + 7\mathbf{w}, \mathbf{u} + 4\mathbf{v}, \mathbf{u} + 7\mathbf{w}\}$ is linearly independent.

[?] 4. The union of two subspaces of a vector space is always a subspace.

? 5. A subset of a spanning set can sometimes form a linearly independent set.