Queen Mary
University of London

Main Examination period 2019

MTH5126: Statistics for Insurance

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

The New Cambridge Statistical Tables 2nd Edition are provided.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiners: D. Boland, S. Liverani

Question 1. [6 marks] An urn B_1 contains 2 white and 3 black balls. A second urn B_2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is black, find the probability that the urn chosen was B_1. It should be assumed that each ball in the selected urn is drawn with equal probability. [**6**]

Question 2. [4 marks]
Consider a general game:

		Player A	
		I	**II**
Player B	**1**	a	b
	2	d	c

(a) Define a **saddle point.** [**2**]

(b) State the conditions for which a saddle point exists in this game. [**2**]

Question 3. [15 marks] The table below shows claims paid on a portfolio of general insurance policies. You may assume that claims are fully run off after three years.

Underwriting year	Development Year			
	0	1	2	3
2012	650	412	217	91
2013	703	489	262	
2014	711	456		
2015	678			

Past claims inflation has been 2% p.a. However, it is expected that future claims inflation will be 4% p.a. Use the inflation adjusted chain ladder method to calculate the outstanding claims on the portfolio. [**15**]

Question 4. [12 marks]
A decision-maker with decreasing risk-aversion has assets of £30 and has a decision problem with the following structure:

	Outcome	
Decision	θ_1	θ_2
d_1	-10	$+5$
d_2	$+15$	-5
Probability	0.3	0.7

The entries in the table represent gains or losses in pounds. For example, with decision d_1 and outcome θ_1 the decision-maker will end up with assets of £20 and with decision d_2 and outcome θ_2 they will end up with assets of £25.
You are given the following excerpt from the decision-maker's utility table:

£x	$u(x)$	£x	$u(x)$
0	0.000	180	0.797
10	0.221	185	0.802
15	0.300	190	0.807
20	0.364	195	0.811
25	0.415	200	0.816
30	0.458	210	0.825
35	0.493	220	0.834
40	0.523	230	0.842
45	0.548	240	0.849
50	0.570	250	0.847

(a) Advise the decision-maker on whether decision d_1 or d_2 is the better course of action. **[7]**

(b) Would your advice remain the same if the decision-maker's assets were £200? **[5]**

You should justify any advice given and show clear workings.

Question 5. [20 marks]

(a) A new insurance policy is sold to a limited number of existing policy holders. The number of claims in the first six months is 18. Suppose the number of claims each month is assumed to have a Poisson distribution with mean θ, independent of all other months. Write down the likelihood function. [**4**]

(b) The prior distribution of θ is given by a Gamma distribution. When designing the policy it was thought that based on the claims history of these policy holders the mean number of claims per month would be 4 claims with variance 1. Show that a Gamma distribution $Gamma(16,4)$ is a suitable prior. [**2**]

(c) Find the posterior distribution of θ. [**4**]

(d) Find the form of the Bayes estimate of θ under squared error loss. [**6**]

(e) Calculate the value of the Bayes estimate. Show it can be written as a weighted average of the prior mean and the data mean and interpret the weights. [**4**]

Question 6. [10 marks] The table below shows aggregate annual claim statistics for three risks over a period of eight years. Annual aggregate claims for risk i in year j are denoted by X_{ij}.

Risk i	$\bar{X}_{i.} = \frac{1}{8}\sum_{j=1}^{8} X_{ij}$	$S_i^2 = \frac{1}{7}\sum_{j=1}^{8}(X_{ij} - \bar{X}_{i.})^2$
1	213.11	411.19
2	91.15	94.23
3	134.23	38.6

(a) What are the assumptions of the Empirical Bayes Credibility Theory (EBCT) Model 1? [**2**]

(b) Calculate the credibility premium of each risk under the assumptions of EBCT Model 1. [**8**]

Question 7. [10 marks] An insurance company has a portfolio of policies under which individual loss amounts follow an exponential distribution with mean λ^{-1}. In one year, the insurer observes 75 claims with mean claim amount 107.8.

(a) Show that the maximum likelihood estimate of λ is 0.0093. [**8**]

(b) Find the probability that an individual loss amount will be greater than 107.8. [**2**]

Question 8. [15 marks]
A manufacturing company is analysing the number of accidents that occur each year on the factory floor. They believe that the number of accidents per year, N, has a geometric distribution with parameter 0.7 so that:

$$P(N=n) = 0.7 \times 0.3^n, \qquad n = 0,1,2,..$$

For each accident i, the number of employees injured is Y_i, where $Y_i = X_i + 1$ and X_i is believed to have a $Poisson(1.8)$ distribution.

The company has taken out an insurance policy, which provides a benefit of £5,000 to each injured employee, up to a maximum of three employees per accident, irrespective of the level of injury. There is no limit on the number of accidents that may be claimed for in a year.

(a) Show that $E(S) = 1.0166$ and $var(S) = 3.6859$, where S is the total number of employees claiming benefit in a year under this policy. **[9]**

(b) Hence find the mean and variance of the aggregate amount paid out under this policy in a year. **[6]**

Question 9. [8 marks]

(a) Explain the difference between **IBNR** and an **Outstanding Claims Reserve.** **[4]**

(b) Explain the difference between **Proportional** and **Non-Proportional** reinsurance and give two examples of each. **[4]**

End of Paper – An appendix of 2 pages follows.

Statistics – Common Distributions

Discrete Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{N}$	$N = 1, 2, \ldots$ $x = 1, 2, \ldots, N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$
Bernoulli	$p^x(1-p)^{1-x}$	$0 \le p \le 1, x = 0, 1$	p	$p(1-p)$
Binomial	$\binom{n}{x}p^x(1-p)^{n-x}$	$0 \le p \le 1, n = 1, 2, \ldots$ $x = 0, 1, \ldots n$	np	$np(1-p)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	$\lambda > 0, x = 0, 1, 2, \ldots$	λ	λ
Geometric	$p(1-p)^x$	$0 < p \le 1, x = 0, 1, 2, \ldots$	$\frac{(1-p)}{p}$	$\frac{(1-p)}{p^2}$

Continuous Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{b-a}$	$-\infty < a < b < \infty$ $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\exp[\frac{-(x-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	μ	σ^2
Lognormal (μ, σ^2)	$\frac{1}{x\sqrt{2\pi\sigma^2}}\exp[\frac{-(logx-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	$e^{(\mu+\frac{1}{2}\sigma^2)}$	$e^{(2\mu+\sigma^2)}(e^{\sigma^2}-1)$
Exponential	$\lambda e^{-\lambda x}$	$\lambda > 0, x \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma (α, λ)	$\frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)}$	$\lambda > 0, \alpha > 0, x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
Weibull (c, γ)	$c\gamma x^{\gamma-1}e^{-cx^\gamma}$	$c > 0, \gamma > 0, x > 0$	$c^{-\frac{1}{\gamma}}\Gamma(1+\gamma^{-1})$	$c^{-\frac{2}{\gamma}}[\Gamma(1+2\frac{1}{\gamma})$ $-\Gamma^2(1+\frac{1}{\gamma})]$
Pareto(α, λ)	$\frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, x > 0$	$\frac{\lambda}{(\alpha-1)}$	$\frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)}$
Burr$(\alpha, \lambda, \gamma)$	$\frac{\alpha\gamma\lambda^\alpha x^{\gamma-1}}{(\lambda+x^\gamma)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, \gamma > 0, x > 0$	Not required	Not required

Useful Formulae

EBCT Model 1

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{(N-1)}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

EBCT Model 2

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2\right]$$

Intermediate calculations

$\sum_{j=1}^{n} P_{ij} = \bar{P}_i$ $\quad \sum_{i=1}^{N} \bar{P}_i = \bar{P}$ $\quad \frac{1}{(Nn-1)}\sum_{i=1}^{N}\bar{P}_i\left(1-\frac{\bar{P}_i}{\bar{P}}\right) = P^*$

$\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}_i} = \bar{X}_i$ $\quad \sum_{i=1}^{N}\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}} = \bar{X}$

End of Appendix.