Main Examination period 2017

MTH6139: Time Series

Duration: 2 hours

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Examiners: L.I.Pettit, N.Rodosthenous

Question 1. [25 marks]

(a) Consider a time series model

$$X_t = m_t + Y_t$$

where the trend m_t is a polynomial function of t with degree k and coefficients $\beta_0, \beta_1, \ldots, \beta_k$. The first difference is defined as

$$\nabla X_t = X_t - X_{t-1}.$$

(i) Show that if m_t is a polynomial function of t with degree 1, then the first difference gives

$$\nabla X_t = \beta_1 + \nabla Y_t.$$

[4]

(ii) Similarly, assume that m_t is a polynomial with degree 2. Find the second difference as a function of the coefficients. **[5]**

(b) (i) For a time series model

$$X_t = m_t + Y_t$$

define a **linear filter**. **[2]**

(ii) What does it mean to say that a linear filter **passes through without distortion**? **[2]**

(iii) A time series is to be smoothed by fitting a quadratic polynomial to successive groups of 5 observations, thus obtaining a weighted moving average filter. Find the filter which passes through without distortion, if least squares fitting is used. **[12]**

Question 2. [21 marks] An MA(1) process with parameter θ is defined by the equation

$$X_t = Z_t + \theta Z_{t-1},$$

where $\{Z_t\}$ is a white noise process, that is, a sequence of uncorrelated random variables with mean zero and constant variance σ^2.

(a) Define what it means for a moving average process to be **invertible**. [**2**]

(b) Show that an MA(1) process is invertible if the parameter θ satisfies a condition which you should state. [**5**]

(c) The autocovariance of X_t and $X_{t+\tau}$ is defined to be $\gamma(\tau)$. For the MA(1) process with parameter θ find $\gamma(0)$ and $\gamma(1)$ and write down $\gamma(\tau)$ for $\tau \geq 2$. Hence calculate the autocorrelation function (ACF) for the MA(1) process. [**7**]

(d) Show that an MA(1) process with parameter θ^{-1} has the same ACF as an MA(1) process with parameter θ. [**2**]

(e) Consider two MA(1) processes with parameters $\theta = 0.25$, $\sigma^2 = 16$ and $\theta = 4$, $\sigma^2 = 1$ respectively. Show that they have the same autocovariance function (ACVF). Explain how you can choose between these processes by considering the invertibility of the processes. [**5**]

Question 3. [15 marks]

(a) Define the sample autocorrelation function (ACF) of a time series with n observations. Explain briefly how the sample partial autocorrelation function (PACF) can be calculated. [**6**]

(b) Describe briefly the expected behaviour of the ACF and PACF for autoregressive (AR(p)), moving average (MA(q)) and autoregressive moving average (ARMA(p, q)) processes. [**6**]

(c) Values of the sample ACF and of the sample PACF for lags $\tau = 1, \ldots, 5$ of an observed time series are given in the following tables. What kind of model is the time series most likely to follow? Explain your answer. [**3**]

ACF					
τ	1	2	3	4	5
$\hat{\rho}(\tau)$	0.9	0.8	0.5	0.2	0.1

PACF					
τ	1	2	3	4	5
$\hat{\phi}_{\tau\tau}$	0.9	0.01	-0.02	0.03	-0.02

Question 4. [29 marks]

(a) Consider the following three time series models where the error terms Z_t are uncorrelated random errors with zero mean and constant variance.

(i)
$$X_t - 0.3X_{t-1} = Z_t.$$

(ii)
$$X_t - 1.2X_{t-1} - 0.2X_{t-2} = Z_t - 0.5Z_{t-1}.$$

(iii)
$$X_t - X_{t-1} = Z_t.$$

For each model, say whether it is stationary or not and specify p and q in the standard ARMA (p, q) framework. **[8]**

(b) Show that the model
$$X_t - X_{t-1} = Z_t.$$
can be written in the ARIMA(p, d, q) framework and specify the values of p, d and q. **[3]**

(c) Consider the time series
$$X_t + \frac{1}{6}X_{t-1} - \frac{1}{3}X_{t-2} = Z_t - \frac{3}{4}Z_{t-1} + \frac{1}{8}Z_{t-2},$$
where Z_t is a white noise random variable.

(i) Determine the values of p and q so that there is no parameter redundancy in the model. **[5]**

(ii) Check whether the process is causal and invertible. **[4]**

(iii) Obtain a linear process form of this time series. **[9]**

Question 5. [10 marks] An autoregressive process $\{X_t\}$ of order 2 has the form

$$X_t = \frac{1}{3}X_{t-1} + \frac{2}{9}X_{t-2} + Z_t,$$

where Z_t is a zero-mean white noise random variable.

(a) The homogeneous difference equations for the autocovariance function γ are given by

$$\gamma(\tau) - \frac{1}{3}\gamma(\tau-1) - \frac{2}{9}\gamma(\tau-2) = 0 \quad \text{for } \tau \geq 2$$

with initial conditions

$$\begin{aligned} \gamma(0) - \frac{1}{3}\gamma(-1) - \frac{2}{9}\gamma(-2) &= \sigma^2 \\ \gamma(1) - \frac{1}{3}\gamma(0) - \frac{2}{9}\gamma(-1) &= 0 \end{aligned}$$

Write down these difference equations as a function of the autocorrelation function $\rho(\tau)$.
Write down $\rho(0)$ and evaluate $\rho(1)$. **[4]**

(b) The general solution of the difference equation for the autocorrelation function is of the form

$$\rho(\tau) = c_1\zeta_1^{-\tau} + c_2\zeta_2^{-\tau},$$

where ζ_1 and ζ_2 are the roots of $\phi(z) = 1 - \frac{1}{3}z - \frac{2}{9}z^2$. Find the autocorrelation function.

Hint: Use part (a). **[6]**

End of Paper.