Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH6116: Design of Experiments

Duration: 2 hours

Date and time: 14th May 2015, 14:30–16:30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.
The New Cambridge Statistical Tables are provided.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it shall be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiner(s): H. Maruri-Aguilar and B. Bogacka

 TURN OVER

Question 1 (24 marks). A new method for making concrete has been proposed. To test whether the new method has improved the material properties of concrete, a number of sample blocks are made by the new and the old method and the compressive strengths in pounds per square inch are measured in each block. The two methods will be compared using a standard double sided t-test which corresponds with the analysis of a completely randomized design.
There is budget for making 15 of these concrete blocks and the technicians in charge have come with two potential designs:

I Using equal replication $r_1 = r_2 = 7$.

II Using all the budget so that $r_1 = 7, r_2 = 8$.

In the above, r_1 are the replications of the standard method while r_2 are replications of the new method.
Advise them by answering the following questions.

(a) One technician tells you that "among all designs, those with equal replication minimize the average variance so design I is better than design II". How do you respond to this? [5]

(b) Explain what is meant by power in a statistical test. [2]

(c) After further questioning, it turns out that the technicians are interested in detecting a difference between the methods of 4 pounds per square inch with probability of at least 80%, while keeping the probability of error of type one equal to 5%. A previous study showed that the variance of a single unit is $\sigma^2 = 7.1$.

Compute the power to detect such change for design I [6]

and design II. [6]

(d) Summarize your conclusions regarding the power of detecting the difference as in Question (c).

[3]

(e) Briefly comment what happens to your analysis and conclusions if the replications of design II are $r_1 = 8$ and $r_2 = 7$ rather than $r_1 = 7$ and $r_2 = 8$. [2]

Question 2 (24 marks).
An experiment performed in Rothamsted in 1935 had the purpose of comparing various types of fumigant, in single and double doses, for their ability to control eelworms in the soil where oats were being grown. A "control" treatment (i.e. no fumigant) was included. The information about treatments and levels of the treatment factors `fumigant`, `dose` and `type` is given in the table below.

treatment	1	2	3	4	5	6	7	8	9
`dose`	0	1	2	1	2	1	2	1	2
`type`	Z	S	S	K	K	M	M	N	N
`fumigant`	1	2	2	2	2	2	2	2	2
Replication Pattern I	16	3	1	6	2	9	3	6	2
Replication Pattern II	16	2	2	6	2	6	6	6	2

(a) List the equivalence classes of factors `fumigant`, `dose` and `type`. **[4]**

(b) Show that factor `fumigant` is equal to the supremum of factors `dose` and `type`. **[5]**

(c) Build the Hasse diagram for the treatment factors. **[5]**

(d) Determine if factors `dose` and `type` are orthogonal for each of the replication patterns I and II given above. To achieve this, use Theorem 10.5 given below. **[10]**

NOTE I

The following is Theorem 10.5, taken from the book by R.A. Bailey.

Theorem 10.5 Factors F and G are orthogonal to each other if and only if the following two conditions are satisfied within each class of $F \vee G$ separately:

(i) every F-class meets every G-class;

(ii) all these intersections have size proportional to the product of sizes of the relevant F-class and G-class.

Question 3 (26 marks). A study was performed to determine the effect of two types of chemical (termed A and B) in tree bark. Six trees were involved in the study; one type of chemical was applied to each tree and two samples of bark were taken from each tree. The amount of rust (milligrams per sample) was measured in each sample. The scheme of the experiment and data are given below. Note that $43^2 + 47^2 + 58^2 + \cdots + 54^2 = 33088$ and $(43 + 47 + 58 + \cdots + 54)^2 = 391876$.

Data						
Tree	1	2	3	4	5	6
Chemical	A	B	A	B	A	B
Rust	43	58	53	60	45	59
	47	57	49	57	44	54

(a) Determine Hasse diagrams for plot factors and for treatment factors. **[7]**

(b) Build a combined Hasse diagram. **[5]**

(c) Give the ANOVA table. Test for the effect of chemical type, and report your conclusions. **[10]**

(d) Give the standard error for comparing the means for the two chemical types. **[4]**

Question 4 (26 marks). The ability of an industrial process to produce strong metal pieces was assessed through an experimental study. The study involved six batches, each of which contained three runs, while each run contained three samples of metal alloy. The strength index of the metal samples was measured, totalling 54 observations. Factors under study were temperature and machining speed. Runs per batch were performed under three different temperatures, and in turn samples per run used three different machining speeds. The following table contains part of the analysis of variance table from `GenStat`, with some information missing replaced by the symbol 'x'.

Variate: strength

Source of variation	d.f.	s.s.	m.s.	VR
batch stratum	x	27350.1	x	
batch.run stratum				
temperature	x	683.8	x	x
Residual	x	9914.9	x	
batch.run.*Units* stratum				
speed	x	17490.3	x	x
speed.temperature	4	1673.5	x	x
Residual	x	9010.9	300.4	
Total	x	66123.4		

(a) Build a Hasse diagram for the plots. [5]

(b) Complete the missing information in the analysis of variance table. [10]

(c) Perform the hypothesis tests for the effects of factors temperature, speed and for their interaction. [7]

(d) Give `GenStat`'s commands needed to analyze the experiment's data. [4]

End of Paper.