QUESTION 1

Not yet answered Marked out of 5.00 Flag question

Determine the fixed point of the function $f : [0,1] \longrightarrow [0,1]$ defined as

$$f(x) = \begin{cases} x + \frac{2}{3} & 0 \le x < \frac{1}{3} \\ \frac{3}{2} - \frac{3}{2}x & \frac{1}{3} \le x \le 1 \end{cases}.$$

Give your answer in decimal form to 5 decimal places, i.e., 0.12345, with appropriate rounding.

Answer:

QUESTION 2

Not yet answered Marked out of 5.00 Flag question

A first-order discrete dynamical system takes on the following states

{3.4, 6.9, 7.3, 5.0, 6.9, ...}

What can be concluded about the system? Select all that apply.

Select one or more:

- a. It has a cycle of period 3
- b. It is a chaotic system in the Devaney sense
- c. It has a cycle of period 2
- d. It has a fixed point
- e. It does not have any k-cycles for k < 5

QUESTION 3

Not yet answered Marked out of 6.00 Flag question

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = -x^{51}$. Fill in the following table:

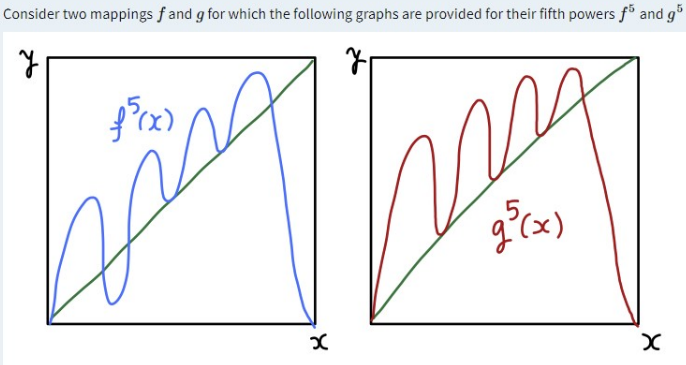
Diffeomorphism?	Order-preserving?	Order-reversing?
Fixed Points?	**Prime period 2 points?**	**Prime period k points (k>2)?**

Yes | No | Not applicable | Not enough information

QUESTION 4

Not yet answered Marked out of 10.00 Flag question

Consider two mappings f and g for which the following graphs are provided for their fifth powers f^5 and g^5



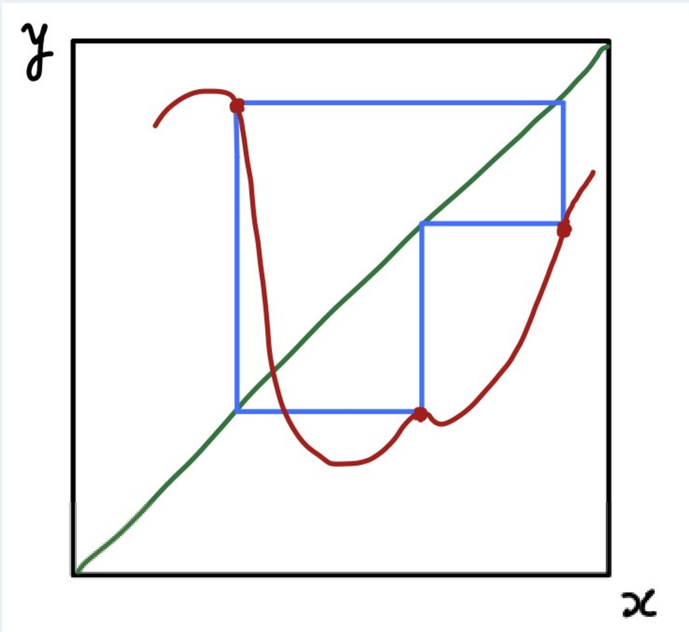
Are f and g topologically conjugate? You may answer yes, no, not enough information, etc. but must provide reasoning for your answer by typing a few brief sentences in the text box below.

Answer:

QUESTION 5

Not yet answered Marked out of 5.00 Flag question

The figure below depits an orbit web for the first 40 iterations of a discrete dynamical system; note that the scale is not given. What specific type of dynamics does the system display? The system exhibits [ ] .



QUESTION 6

Not yet answered Marked out of 5.00 Flag question

Consider the discrete dynamical system $x_{n+1} = f(x_n)$ where $f(x) = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 1$. Which of the statements below are correct?

- a. The point $x_0 = 0$ is a repelling fixed point.
- b. The point $x_0 = 0$ generates an attracting 2-cycle.
- c. The point $x_0 = 0$ generates a repelling 2-cycle.
- d. The point $x_0 = 0$ generates an attracting 3-cycle.
- e. The point $x_0 = 0$ is an attracting fixed point.
- f. The point $x_0 = 0$ generates a repelling 3-cycle.
- g. The point $x_0 = 0$ is neither a fixed point nor does it generate a periodic orbit.

QUESTION 7

Not yet answered Marked out of 5.00 Flag question

For a continuous function $f : \mathbb{R} \longrightarrow \mathbb{R}$, which of the following prime period orbits is possible without the other three?

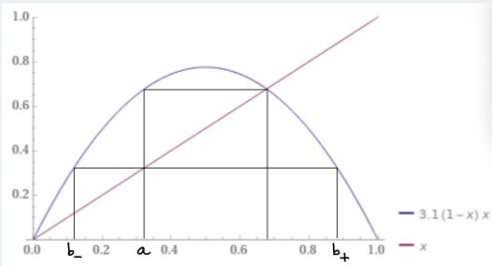
24, 28, 64, 80

Answer:

QUESTION 8

Not yet answered Marked out of 15.00 Flag question

The logistic map $f(x) = 3.1x(1-x)$ has one nonzero fixed point in [0,1]. Use the partially annotated diagram below to argue that f has infinitely many preperiodic points.



QUESTION 9

Not yet answered Marked out of 5.00 Flag question

The shift map σ on the space S of all infinite binary sequences has 30 disjoint prime period 8 cycles.

Is this statement true or false?

Select one:

- True
- False

QUESTION 10

Not yet answered Marked out of 8.00 Flag question

Consider the Doubling Map $f(x) = 2x \bmod 1$ which has six prime period 5 orbits. In S, the space of binary sequences, these orbits are generated by $\overline{10000}, \overline{11000}, \overline{11100}, \overline{11110}, \overline{10101}, \overline{01010}$.

Use this information to determine the subinterval of [0,1] in which each of the 5-cycles of f listed below resides.

The associated 5-cycle of f generated by $\overline{11000}$	Choose...
The associated 5-cycle of f generated by $\overline{11110}$	Choose...

QUESTION 11

Not yet answered Marked out of 5.00 Flag question

Consider the mappings $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = -7x^3$ and $g(x) = x^3$, respectively. Then f is topologically conjugate to g via a homeomorphism of the form $h(x) = \alpha x + \beta$ for some $\alpha, \beta \in \mathbb{R}$.

Is this statement true or false?

Select one:

- True
- False

QUESTION 12

Not yet answered Marked out of 5.00 Flag question

Consider the "middle fifths" Cantor set C_5 obtained from the unit interval [0,1] by first removing the middle fifth of the interval [0,1] (the open interval $\left(\frac{2}{5}, \frac{3}{5}\right)$), then the middle fifth of each of the two remaining intervals etc. What is the total length of the set remaining at the n^{th} stage?

Select one:

- a. $\frac{4^{n-1}}{5^{n-1}}$
- b. $\frac{4^n}{5^n}$
- c. $\frac{4^{n-1}}{5^{n+1}}$
- d. $\frac{4^{n-1}}{5^n}$

QUESTION 13

Not yet answered Marked out of 15.00 Flag question

In your own words, briefly explain the relationship between Lyapunov exponents and chaos for a discrete dynamical system.

QUESTION 14

Not yet answered Marked out of 6.00 Flag question

For this problem we will consider a set S in $\mathbb{R}^2$ constructed via a recursive process similar to the Sierpinski Carpet. We begin in the initial Step 0 with the unit square $[0,1] \times [0,1]$. In Step 1 we subdivide the original square into nine equal squares and remove the upper middle square and the bottom left two squares. In Step 2, we subdivide each of the remaining squares into nine equal squares and remove the upper middle squares and the bottom left two squares. In general, going from Step n to Step n+1 we subdivide each of the remaining squares into nine equal squares and remove the upper middle two squares and the bottom left two squares. S is the set that is formed by the intersection of all sets from each stage. Below we picture Step 1 through Step 4 in the construction of S.

The Box Counting dimension of S is [ ] so the set [ ] .