



MTH5110: Introduction to Numerical Computing

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Duration: 2 hours

Surname/family name:	
First name/given name:	
Student number:	

You should attempt ALL questions. Marks available are shown next to the questions.

Answer each question in the corresponding subsection headed "Answer to Problem #" (where # is the problem number), using the Maple input or text regions as appropriate. You may also add your own Maple input or text regions, but you should keep each answer in the corresponding subsection. You must use Maple to perform all calculations. Do not delete any relevant input or output; you will score marks only for what is visible in the document you submit. There may be more than one correct solution to each question; any working solution will be accepted provided it satisfies the requirements of the question.

This exam is open book. You may access any information you want, but must work entirely by yourself. You may not communicate, nor attempt to communicate, with anyone else, nor solicit assistance in any way. Please be aware that details of all internet activity on your computer may be logged. You may do rough work on your own paper, which will not be collected by the invigilators. A mobile phone that causes a disruption in the exam is an assessment offence.

Calculators may be used in this examination.

Examiners: T. Popiel, F. J. Wright

Do not read anything below this line until instructed to do so by an invigilator.

Problem 1 [26 marks]

(a) [6 marks]

Compute a floating-point approximation of π in 6-digit precision. Compute also the relative error in this approximation.

Answer to Problem 1(a)

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(b) [8 marks]

Compute $\sqrt{\sqrt{5.0} \cdot \sqrt{5.0} - 5.0}$ and explain why the result seems to be a complex number.

Answer to Problem 1(b)

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(c) [12 marks]

The *Padovan sequence* is the sequence of integers defined by $P(0) = P(1) = P(2) = 1$, and $P(n) = P(n-2) + P(n-3)$ for $n \geq 3$. Write a procedure that takes as input a non-negative integer n and returns the value of $P(n)$. Use your procedure to compute $P(21)$.

Answer to Problem 1(c)

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Problem 2 [22 marks]

(a) [10 marks]

Plot both of the functions $f(x) = x^3$ and $g(x) = (x-1)^2 \cdot \cos(x)$ together on a single picture, for $x \in [-3, 3]$. Using the Newton-Raphson method, define an iteration map that could be used to compute the point at which the graphs of $f(x)$ and $g(x)$ intersect. You should explain your reasoning, but you are *not* required to compute the point of intersection.

Answer to Problem 2(a)

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(b) [12 marks]

Recall that the *inverse* of a function $h(x)$ is the function $h^{-1}(y)$ with the property that $h^{-1}(y) = x$ if and only if $h(x) = y$. Consider the function $h(x) = x - 0.5 \cdot \sin(x)$. Write a procedure that takes as input a floating-point number y and approximates $h^{-1}(y)$ using the Newton-Raphson method with absolute error at most 10.0^{-5} . Use your procedure to compute an approximation to $h^{-1}(2.5)$.

Answer to Problem 2(b)

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Problem 3 [28 marks]

(a) [10 marks]

Write a procedure that computes a root of a function via the bisection method. Your procedure should take as input a function $f(x)$, the endpoints of an interval $[a, b]$ for which either $f(a) < 0$ and $f(b) > 0$ or $f(a) > 0$ and $f(b) < 0$, and an error threshold ϵ . The absolute error in the output should be at most ϵ .

Answer to Problem 3(a)

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(b) [10 marks]

Plot the function $f(x) = x^2 \cdot \exp(x) - \frac{1}{2}$ over the interval $[-4, 1]$. Use your procedure with error threshold $\epsilon = 10.0^{-10}$ to compute the three solutions of $f(x) = 0$.

Answer to Problem 3(b)

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(c) [8 marks]

Suppose that the bisection method has been performed on some function with initial interval $[1.5, 4.25]$, and has returned the value 3.21875 . Explain the bisection steps that must have been performed in order to obtain this output, and determine an upper bound for the associated absolute error.

Answer to Problem 3(c)

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Problem 4 [24 marks]

(a) [10 marks]

Write a procedure that uses the trapezoidal rule to compute an approximation to the definite integral

$$\int_0^{2 \cdot \pi} \frac{1}{3 + 2 \cdot \cos(x)} dx.$$

Your procedure should have a single input, the number of sub-intervals n .

Answer to Problem 4(a)

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(b) [8 marks]

Produce a plot that shows the absolute error in your approximation for $n = 1, 2, \dots, 25$. Explain how the absolute error depends on n . (Hint: you may wish to load the *plots* package by running the command `with(plots):`, and then use the *listplot* command to produce your plot.)

Answer to Problem 4(b)

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(c) [6 marks]

The integral in part (a) is exactly equal to $\frac{2 \cdot \pi}{\sqrt{5}}$. Use your procedure to approximate $\frac{2 \cdot \pi}{\sqrt{5}}$ with absolute error less than 10.0^{-8} . How many sub-intervals are required? Justify your answer.

Answer to Problem 4(c)

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