

MTH4110: Mathematical Structures

Duration: 2 hours

Date and time: 3rd May 2016, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it shall be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiner(s): O. F. Bandtlow

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Question 1. Let $n \in \mathbb{N}$. Using induction, prove that

$$(1-x)\sum_{k=0}^{n-1} x^k = 1 - x^n$$

for any real number *x*.

[12]

Question 2.

(a) Give the definition of a *prime number*.

[4]

(b) Given any natural number n is there a prime number larger than n^n ? Give reasons for your answer. You are allowed to quote results proved in the lectures.

[4]

(c) Let n be a natural number. Prove, using the equation in Question 1 or otherwise, that if $2^n - 1$ is prime, then n is prime.

[8]

Question 3. Let A, B, and C be sets.

(i) injective,

(a) Explain what is meant for A to be a *subset* of B.

[4]

(b) Show that if $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$, then A = B = C.

[4]

(c) Is it possible that $A \in B$ and $A \subseteq B$? Give reasons for your answer.

(ii) surjective,

[4]

Question 4.

(a) Let A and B be sets, and let $f: A \rightarrow B$ be a function. Explain what is meant by saying that f is

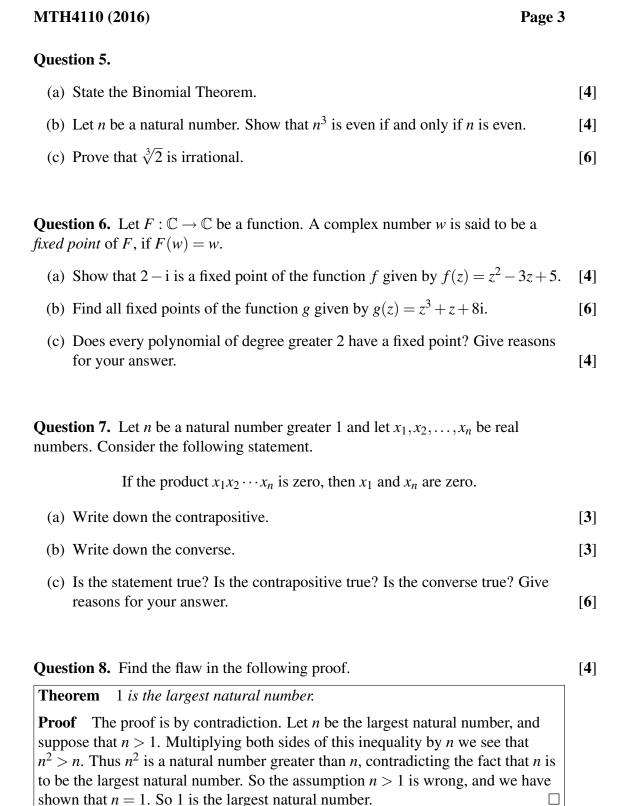
(iii) bijective. [6]

(b) Is there a bijection from \mathbb{Z} to \mathbb{R} ? Give a brief explanation for your answer.

[4]

(c) Let $\mathscr{F}(\mathbb{N})$ denote the set of all finite subsets of \mathbb{N} and let the function $f: \mathscr{F}(\mathbb{N}) \to \mathbb{N}$ be given by $f(A) = |\mathscr{P}(A)|$, where $\mathscr{P}(A)$ denotes the power set of A. Is f injective? Is it surjective? Give reasons for your answers.

[6]



End of Paper.