Queen Mary University of London

Main Examination period 2023 – January – Semester A

MTH4107 / MTH4207: Introduction to Probability

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

The exam is intended to be completed within **2 hours**. However, you will have a period of **3 hours** to complete the exam and submit your solutions.

You should attempt ALL questions. Marks available are shown next to the questions.

The exam is closed-book, and **no outside notes are allowed**.

Only approved non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any unauthorised notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: J. Moriarty, V. Fain

Questions 1 to 12 should be answered on the multiple choice answer sheet provided, using an HB pencil. Question 13 should be answered in the answer book.

Question 1 [6 marks]. You roll a six-sided die. If the die shows a "one", you toss a coin three times. What is the size of the sample space?

(a) 24

(b) 13

(c) 37

(d) 8

[**6**]

Question 2 [6 marks]. Suppose A and B are events. Which of the following probabilities has the same value as $1 - \mathbb{P}(A^c \cup B)$?

(a) $\mathbb{P}(\mathcal{S} \triangle A)$

(b) $\mathbb{P}(A \setminus B)$

(c) $\mathbb{P}(B|A)\mathbb{P}(A)$

(d) $\mathbb{P}(A \cup B)$

[**6**]

Question 3 [6 marks]. You want to contact a friend with an urgent message, and decide to try messaging, calling and emailing them. They will answer the call with probability 0.4, will read the message with probability 0.6, and will read the email with probability 0.5. What is the probability that your friend gets your urgent message? You may assume that the events involved are mutually independent.

(a) 0.12

(b) $\frac{1}{3}$

(c) 0.88

(d) 0.72

(e) None of the other choices

Question 4 [6 marks]. In the new sport of Ultimate Aerobie, a squad consists of five defenders and seven forwards. A team consists of two defenders and three forwards. How many different teams can be selected from one squad?

(a) 85

(b) 350

(c) 792

(d) 81600

(e) None of the other choices

Question 5 [8 marks]. In an Ultimate Aerobie squad of five defenders and seven forwards there are two identical twins. One twin is a defender and the other is a forward. If the team of two defenders and three forwards is selected at random from the squad, what is the probability that both twins are selected? [Available answers are given to two decimal places.]

(a) 0.08

(b) 0.12

(c) 0.20

(d) 0.17

(e) None of the other choices

Question 6 [6 marks]. Let E_1, E_2, and E_3 be events for which the condition $\mathbb{P}(E_i \cap E_j) = \mathbb{P}(E_i)\mathbb{P}(E_j)$ holds for each $i, j \in \{1, 2, 3\}$ with $i \neq j$. What can be said about these events?

(a) They are mutually independent but not pairwise independent

(b) They are pairwise independent but not mutually independent

(c) They are mutually independent but not necessarily pairwise independent

(d) They are pairwise independent but not necessarily mutually independent

(e) None of the other choices

Question 7 [6 marks]. In a group of people, every person uses at least one of the social media services WitzApp, BookFace or WhoTube. A person from the group is selected at random. Which of the following sets of events partition the sample space?

(a) "Uses WitzApp", "Does not use Witzapp but uses BookFace and WhoTube"

(b) "Uses Bookface but not WhoTube", "Uses WitzApp or Bookface"

(c) "Uses WitzApp", "Uses either BookFace or WhoTube"

(d) None of the other choices

Question 8 [6 marks]. When logging in to your internet bank, a random authentication number X is sent to your mobile phone to confirm your identity. You have noticed that X is always even. If a is a positive constant and the distribution of the random variable aX is Geometric, what is a?

(a) 1/2

(b) 1

(c) 2

(d) Cannot be determined without more information

(e) None of the other choices

Question 9 [8 marks]. In London each day is sunny with the same probability p, independently of all other days. You decide to count the number of days X until the next sunny day, and you notice that the variance of X is 3.75. What is the value of p? [Answers are given to one decimal place]

(a) 0.3

(b) 0.6

(c) 0.5

(d) 0.4

(e) None of the other choices

Question 10 [8 marks]. In a multiple choice exam there are five possible answers for each question. You decide to adopt a probabilistic strategy, choosing each answer at random. To account for such strategies, the examiner subtracts N marks from the score (where N is a positive constant) so that the average grade under this random strategy is 0. If there are 10 questions, what is the variance of your total score?

(a) $\frac{3}{4}$

(b) $10 - N$

(c) $\frac{8}{5}$

(d) $\frac{1}{5}$

(e) None of the other choices

Question 11 [6 marks]. Suppose the probability that you receive a message during any given minute is 1%, independently of what happens at other times. Let X be the number of messages you receive per hour. What would be a reasonable approximation to the distribution of X?

(a) Geometric with parameter 0.6

(b) Bernoulli with parameter 0.01

(c) Poisson with parameter 0.6

(d) Geometric with parameter 0.01

(e) Poisson with parameter 60

Question 12 [8 marks].

Suppose that X and Y are discrete random variables with $\mathrm{Cov}(-2X, Y+4) = -1$, $\mathbb{E}(XY) = 1$, $\mathbb{E}(X) = 3$, $\mathrm{Var}(X) = 2$ and $\mathrm{Var}(Y) = 3$. What is $\mathrm{Corr}(X, Y)$ to two decimal places?

(a) 0.50

(b) Cannot be determined

(c) 0.33

(d) 0.20

(e) None of the other choices

[8]

Question 13 [20 marks].

(a) Prove that, if the random variables U and V are independent, then $\mathbb{E}\left(\frac{U}{V}\right) = \mathbb{E}(U)\, E\left(\frac{1}{V}\right)$. **[5]**

(b) Let X and Y be random variables which each take values from the set $\{1,2\}$ and have joint probability mass function

$$\mathbb{P}(X = x, Y = y) = a((x-1)^2 + (y-1)^2),$$

where a is a constant.

(i) Write the joint probability mass function in the form of a table and determine the constant a. **[6]**

(ii) Are X and Y independent? Justify your answer. **[2]**

(iii) Let Z be a random variable with the Binomial(5, 0.1) distribution, and suppose that X and Z are independent. Calculate $\mathbb{E}\left(\frac{Z}{X}\right)$. **[7]**

End of Paper.