Queen Mary
University of London

Main Examination period 2019

MTH4107 / MTH4207: Introduction to Probability

Duration: 2 hours

Student number | | | | | | | | | | **Desk number** | | | |

Make and model of calculator used _____

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more space use the additional sheets attached at the end of the exam paper.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

This is an OPEN BOOK exam

permitted:	any printed material, e.g. books any handwritten notes photocopies of any kind
prohibited:	using communication devices, e.g. laptops or mobile phones sharing material with other students

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL.

If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it shall be treated as an assessment offence. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: W. Just

This page is for marking purposes only.

Do not write on it.

Question	Mark	Comments
1	/ 25	
2	/ 25	
3	/ 25	
4	/ 25	
Total		

Question 1. [25 marks] You roll a fair die. If you roll a 1 you roll the die once again.

(a) Write down the sample space for this experiment. **[5]**

(b) Are the outcomes of the experiment equally likely? Justify your answer. **[6]**

(c) State the events "you roll at least one even number" and "you roll at least one 1" as subsets of the sample space. **[4]**

(d) Compute the probabilities of the two events of part (c). State the results as irreducible fractions. **[4]**

(e) Compute the conditional probability that "you roll at least one even number" assuming that "you roll at least one 1". State your result as an irreducible fraction. Are the two events in part (c) independent? Justify your answer. **[6]**

Question 2. [25 marks] A bag contains 5 gold, 3 silver and 2 copper coins. You pick coins at random without replacement.

(a) You pick 3 coins. Compute the probability that you pick gold coins only. State the result as an irreducible fraction. **[4]**

(b) You pick 3 coins. Compute the probability that your pick results in one gold, one silver and one copper coin. State the result as an irreducible fraction. **[6]**

(c) You pick 3 coins. Compute the probability that your first pick is a gold coin, the second a silver coin, and the final a copper coin. State the result as an irreducible fraction. **[4]**

(d) You pick 4 coins. Compute the probability that the first two coins are one gold and one silver coin, and the final 2 coins are again one gold and one silver coin. State the result as an irreducible fraction. **[6]**

(e) You pick 2 coins. Compute the probability that the second coin you pick is a silver coin. State the result as an irreducible fraction. **[5]**

Question 3. [25 marks] A bag contains two fair coins and a biased coin. The biased coin has probability $1/4$ coming up head. You pick coins from the bag without replacement and toss the coins once.

(a) You pick one coin and toss it. Compute the probability that the coin comes up head. State the result as an irreducible fraction. **[5]**

(b) Assume the coin in part (a) did come up head. Compute the probability that you have picked a fair coin. State the result as an irreducible fraction. **[5]**

(c) You pick a coin. You pick a second coin and toss it. Compute the probability that the coin comes up head. State the result as an irreducible fraction. **[5]**

(d) You pick a coin and toss it. You pick a second coin and toss it. Compute the joint probability that the toss of the first coin comes up head and the toss of the second coin comes up head. **[5]**

(e) You pick a coin and toss it. You pick a second coin and toss it. One coin is left in the bag. Compute the probability that the remaining coin is fair. State the result as an irreducible fraction. **[5]**

Question 4. [25 marks] Consider two random variables X and Y.

(a) The random variable X takes two values, the value 0 with probability $1/2$ and the value 1. Compute the probability mass function of X, the expectation of X and the variance of X. State the results as irreducible fractions. **[6]**

(b) The random variable Y takes two values, the value -1 with probability $1/4$ and the value 1. Compute the probability mass function of Y, the expectation of Y and the variance of Y. State your results as irreducible fractions. **[6]**

(c) Assume that the joint probability of X and Y both taking the value one is $1/4$. Compute the joint probability mass function of the two random variables. **[4]**

(d) Using the result of part (c), or otherwise, compute the covariance and the correlation coefficient of the two random variables. State the results as irreducible fractions. [**5**]

(e) Are the random variables independent ? Give a reason. **[4]**

Additional space for answers

Write your solutions here

Write your solutions here

Write your solutions here

Write your solutions here

Write your solutions here

End of Paper.