Queen Mary
University of London

MTH4105 Introduction to Mathematical Computing

Duration: 2 hours

Date and time: Thursday 2 June 2016, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators may be used in this examination.

Answer each question in the appropriate subsection headed *Answers* in the Exam answer template provided in QMplus. You must use Maple to perform all calculations. Do not delete any relevant input or output; you will score marks only for what is visible in the document you submit. There may be more than one correct solution to each question; any working solution will be accepted provided it satisfies the requirements of the question.

This exam is open book. You may access any information you want, but must work entirely by yourself. You may not communicate, nor attempt to communicate, with anyone else, nor solicit assistance in any way. Please be aware that details of all internet activity on your computer may be logged. You may do rough work on your own paper, which will not be collected by the invigilators. A mobile phone that causes a disruption in the exam is an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: F. J. Wright, J. Starke

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Question 1 [12 marks]

a) Assign the polynomial $x^3 - 2x$ to the variable y and plot the graph of y against x for $-2 \leq x \leq 2$. **[3 marks]**

b) Compute the exact x values at which y has a (local finite) maximum or minimum. **[3 marks]**

c) Compute the exact y value, y_0, of the (local finite) maximum. **[3 marks]**

d) Compute the amount by which y_0 differs from 1 as an explicit positive number displayed with exactly 5 significant digits (which may require some experimentation). **[3 marks]**

Question 2 [12 marks]

Each of the following expressions contains 5 main elements. Construct each expression by evaluating input that uses an appropriate function such as *seq*, *add* or *mul* and could construct an expression of the form shown containing an arbitrary number of main elements.

a) the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$ **[3 marks]**

b) the set of sets $\{\{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 4, 5\}\}$ **[3 marks]**

c) the polynomial $5x^5 + 4x^4 + 3x^3 + 2x^2 + x$ **[3 marks]**

d) the polynomial $(x-1)(x-2)^2(x-3)^3(x-4)^4(x-5)^5$ **[3 marks]**

Question 3 [12 marks]

a) Assign to the variable A the set of even integers from 0 to 100 inclusive. Use A to assign to the variable B the set whose elements are each half an element of A, i.e. the set $\left\{\frac{r}{2} : r \in A\right\}$. Use the variables A and B in your answers to the rest of this question. **[4 marks]**

b) Enter and evaluate a single expression that evaluates to *true* if A is not a subset of B and B is not a subset of A, and to *false* otherwise. **[3 marks]**

c) Enter and evaluate a single expression that evaluates to *true* if the elements of the set B minus the set A are all odd, and to *false* otherwise. **[5 marks]**

Question 4 [14 marks]

a) The problem is to compute the smallest $n \in \mathbb{N}$ such that $\sum_{i=1}^{n} p_i \geq N$ for some specified $N \in \mathbb{N}$, where p_i denotes the i^{th} prime. Write Maple code to solve this problem for $N = 1000$ as follows. Assign the value 1000 to the variable N. Then use a **do** statement (loop) to compute the sum of successive primes starting from the first and stop the loop as soon as the sum is greater than or equal to N. Display the solution in the form $nmin(N) = n$ (with N and n replaced by their values) **and nothing else**. **[10 marks]**

b) Confirm your result by computing $\sum_{i=1}^{n-1} p_i$ and $\sum_{i=1}^{n} p_i$ directly without using an explicit loop, where n is the value of $nmin(1000)$ that you computed in part (a). **[4 marks]**

Question 5 [12 marks]

Let $A = \{1, 2, 4, 8\}$ and let $B = \{0, 1, 2, 3\}$.

a) Define the formula-based function $f : A \to B$ such that $x \mapsto \log_2(x)$. **[2 marks]**

b) Define the table-based function $g : B \to A$ such that $0 \mapsto 1$, $1 \mapsto 2$, $2 \mapsto 4$, $3 \mapsto 8$. **[2 marks]**

c) Enter and evaluate **a single expression** that evaluates to *true* if the functions f and g are inverses of each other and to *false* otherwise. **[4 marks]**

d) Plot together on the same axes the graphs (as isolated points) of the two functions f and g with an appropriate legend. **[4 marks]**

Question 6 [12 marks]

a) Assign to the variable P the powerset of the set $\{1, 2, 3, 4, 5\}$. **[3 marks]**

b) Show that the subset relation ($\subseteq$) is reflexive and transitive but not symmetric on the domain P by evaluating three appropriate expressions that evaluate to *true* or *false*. **[9 marks]**

Question 7 [14 marks]

a) A function f is defined recursively by $f(0) = 0$, $f(n) = \dfrac{1}{n + f(n-1)}$ for $n > 0$.

Implement this function in Maple as a recursive procedure called f that takes one argument, n, which you can assume to be a non-negative integer. Use procedure f to compute $f(1)$, $f(5)$, $f(10)$. **[7 marks]**

b) A recursive sequence is defined by $f_0 = 0$, $f_n = \dfrac{1}{n + f_{n-1}}$ for $n > 0$. Write a procedure called F that takes one argument, n, which you can assume to be a non-negative integer. Procedure F should compute the elements of the recursive sequence $(f_r : 0 \leq r \leq n)$ for increasing values of r and return the last element computed, namely f_n, by initializing a local variable f to 0 and then within a **do** statement (loop) repeatedly assigning $\dfrac{1}{r+f}$ to f for values of r running from 1 to n. Declare local variables as appropriate. (You are advised not to use subscripted variable names.) Use procedure F to compute $F(1)$, $F(5)$, $F(10)$. **[7 marks]**

Question 8 [12 marks]

a) Compute and assign to the variable S the **set** of exact (complex) solutions of the equation $z^3 - 1 = 0$, and plot the solutions as **isolated points** on an Argand diagram. **[4 marks]**

b) Compute the sum of the elements of S, the set of moduli of the elements of S, and the set of arguments of the elements of S. **[4 marks]**

c) Compute the set of squares of the elements of S, simplify the result to have explicit real and imaginary parts, and show that the resulting set of squares is identical to S by entering and evaluating an appropriate expression that evaluates to *true*. **[4 marks]**

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