Student: ____________________ Date: ____________________	Instructor: Rainer Klages Course: MTH4101/MTH4201 Calculus II 2022	Assignment: Late-summer final assessment 2022

1. Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = 7x^2 - 2x^3 + y^2 + 2xy$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local maximum occurs at ☐.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local maximum value(s) is/are ☐.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local minimum occurs at ☐.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local minimum value(s) is/are ☐.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A saddle point occurs at ☐.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no saddle points.

ID: 13.7.15

2. **(a)** Find the series' radius and interval of convergence. Find the values of x for which the series converges **(b)** absolutely and **(c)** conditionally.

$$\sum_{n=1}^{\infty} \frac{(2x-3)^{2n+1}}{n^{3/2}}$$

(a) The radius of convergence is ☐.
(Simplify your answer.)

Determine the interval of convergence. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The interval of convergence is ☐.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges only at x = ☐. (Type an integer or a simplified fraction.)
- **C.** The series converges for all values of x.

(b) For what values of x does the series converge absolutely?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges absolutely for ☐.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges absolutely at x = ☐. (Type an integer or a simplified fraction.)
- **C.** The series converges absolutely for all values of x.

(c) For what values of x does the series converge conditionally?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges conditionally for ☐.
  (Type a compound inequality. Simplify your answer. Use integers or fractions for any numbers in the expression.)
- **B.** The series converges conditionally at x = ☐.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- **C.** There are no values of x for which the series converges conditionally.

ID: 9.7.31

3. Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = \ln\left(1 + \frac{6}{n}\right)^n$

- **A.** The sequence converges to 0.
- **B.** The sequence converges to **ln** 6.
- **C.** The sequence converges to 6.
- **D.** The sequence diverges.

ID: 9.1-33

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4. Find the average value of the function f over the given region.

$f(x,y) = 7x + 2y$ over the square $0 \le x \le 1$, $0 \le y \le 1$.

- **A.** 9
- **B.** 8
- **C.** $\frac{11}{2}$
- **D.** $\frac{9}{2}$

ID: 14.3-16

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5. Find the derivative of the function at the given point in the direction of **A**.

$f(x,y) = -7x^2 + 2y$, $(-9,-8)$, $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$

- **A.** 74
- **B.** $\frac{622}{5}$
- **C.** $\frac{748}{5}$
- **D.** $\frac{496}{5}$

ID: 13.5-8

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6. Find the limit of f as $(x,y) \to (0,0)$ or show that the limit does not exist. Consider converting the function to polar coordinates to make finding the limit easier.

$f(x,y) = \cos\left(\frac{x^3 - y^3}{x^2 + y^2}\right)$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{(x,y)\to(0,0)} f(x,y) =$ ☐ (Simplify your answer.)
- **B.** The limit does not exist.

ID: 13.2.66

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7. Use any method to determine if the series converges or diverges. Give reasons for your answer.

$\sum_{n=1}^{\infty} \frac{(-7)^n}{4^n}$

Select the correct choice below and fill in the answer box to complete your choice.

- **A.** The series diverges because the limit used in the Ratio Test is ☐.
- **B.** The series converges because it is a geometric series with r = ☐.
- **C.** The series diverges because it is a p-series with p = ☐.
- **D.** The series converges per the Integral Test because $\int_1^{\infty} \frac{1}{4^x}\,dx =$ ☐.

ID: 9.5.24

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8. Evaluate the double integral over the given region R.

$\iint_R yx \sin x \, dA$ R: $-4 \le y \le 4,\ 0 \le x \le \pi$

$\iint_R yx \sin x \, dA =$ [ ]

(Simplify your answer.)

ID: 14.1.19

9. Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by f at a.

$f(x) = e^{8x}$, $a = 0$

$P_0(x) =$ [ ] (Simplify your answer.)

$P_1(x) =$ [ ]

$P_2(x) =$ [ ]

$P_3(x) =$ [ ]

ID: 9.8.1

10. Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2 + y^2}}\, dx\, dy$$

Change the Cartesian integral into an equivalent polar integral.

$\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2 + y^2}}\, dx\, dy = \int_0^{[\ ]} \int_0^{[\ ]} [\ ]\, dr\, d\theta$

(Type exact answers, using π as needed.)

The value of the double integral is [ ].

(Type an exact answer, using π as needed.)

ID: 14.4.19

11. Determine whether the series $\sum_{n=0}^{\infty} \left(\frac{5}{\sqrt{29}}\right)^n$ converges or diverges. If it converges, find its sum.

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The series converges because $\lim_{n\to\infty} \left(\frac{5}{\sqrt{29}}\right)^n = 0$. The sum of the series is [ ].
  (Type an exact answer, using radicals as needed.)
- **B.** The series diverges because $\lim_{n\to\infty} \left(\frac{5}{\sqrt{29}}\right)^n \ne 0$ or fails to exist.
- **C.** The series diverges because it is a geometric series with $|r| \ge 1$.
- **D.** The series converges because it is a geometric series with $|r| < 1$. The sum of the series is [ ].
  (Type an exact answer, using radicals as needed.)

ID: 9.2.53

12. Find the equation for **(a)** the tangent plane and **(b)** the normal line at the point $P_0(1,1,e)$ on the surface $2x \ln y + y \ln z = x$.

(a) Using a coefficient of 3 for y, the equation for the tangent plane is [ ].

(b) Find the equations for the normal line. Let $x = 1 - t$.

$x =$ [ ], $y =$ [ ], $z =$ [ ]

(Type expressions using t as the variable.)

ID: 13.6.9

13. Evaluate $\frac{dw}{dt}$ at $t=5$ for the function $w(x,y)=e^{y}-\ln x;\ x=t^{2},\ y=\ln t$.

- A. $\frac{4}{5}$
- B. $-\frac{3}{5}$
- C. $\frac{3}{5}$
- D. 3

ID: 13.4-3

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14. Use the transformation $u=x+y$, $v=y-x$ to evaluate the given integral by first writing it as an integral over a region G in the uv-plane.

$$\int_0^1 \int_y^{2-y} (x+y)\,e^{(y-x)}\,dx\,dy$$

$$\int_0^1 \int_y^{2-y} (x+y)\,e^{(y-x)}\,dx\,dy = \square$$

(Type an exact answer.)

ID: 14.8.13

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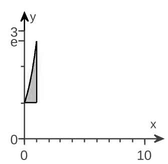
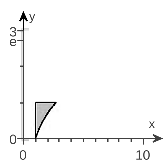
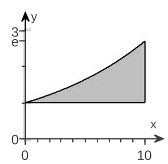
15. Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

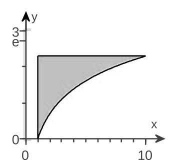
$$\int_0^1 \int_1^{e^x} 9\,dy\,dx$$

Choose the correct graph below.

A.


B.


C.


D.


What is an equivalent double integral with the order of integration reversed?

$$\int_{\square}^{\square} \int_{\square}^{\square} 9\,dx\,dy$$

ID: 14.2.37

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16. Find a formula for the nth term of the sequence.

2, 9, 28, 65, 126, ...

Determine the sequence's formula in terms of n.

$a_n = \square,\ n \ge 1$

ID: 9.1.19

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17. For the function $f(x)=\sqrt{x+49}$, find the Taylor series generated by f at $x=0$.

Choose the correct answer below.

- A. $7+\sum_{n=1}^{\infty} \frac{(-1)^{n}\cdot 1\cdot 3\cdot 5\cdot \cdots \cdot (2n-3)\cdot 49^{(1-2n)}x^{n}}{2^{n}\cdot n!}$
- B. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\cdot 1\cdot 3\cdot 5\cdot \cdots \cdot (2n+1)\cdot 7^{(3-2n)}x^{n}}{2n\cdot n!}$
- C. $7+\frac{1}{14}x+\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\cdot 1\cdot 3\cdot 5\cdot \cdots \cdot (2n-3)x^{n}}{2^{n}\cdot 7^{(2n-1)}\cdot n!}$
- D. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\cdot 1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)\cdot 7^{(-1-2n)}x^{n}}{2^{n}\cdot n}$

ID: 9.8.34

18. Does the series below converge or diverge? Give a reason for your answer. (When checking your answer, remember there may be more than one way to determine the series' convergence or divergence.)

$$\sum_{n=-10}^{\infty} \frac{-6}{n+11}$$

Does the series converge or diverge? Why or why not?

- A. The series diverges. This is revealed by the integral test.
- B. The series converges. This is revealed by the integral test.
- C. The series converges. This is revealed by rewriting the series as a geometric series with $|r| < 1$.
- D. The series diverges. This is revealed by the nth-term test.

ID: 9.3.25

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19. For the given function, complete parts **(a)** through **(f)** below.

$$f(x,y) = \ln\left(x^2 + y^2 - 49\right)$$

(a) Find the function's domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The domain is all points (x,y) satisfying ☐ ≤ 0.
- B. The domain is all points (x,y) satisfying ☐ < 0.
- C. The domain is all points (x,y) satisfying ☐ ≥ 0.
- D. The domain is all points (x,y) satisfying ☐ > 0.
- E. The domain is the entire xy-plane.

(b) Find the function's range. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The range is ☐. (Type your answer in interval notation.)
- B. The range is all real numbers.

(c) Describe the function's level curves. Choose the correct answer below.

- A. For $f(x,y) = 0$, the level curve is the origin. For $f(x,y) \ne 0$, the level curves are ellipses centered at the origin and major and minor axes along the x- and y-axes, respectively.
- B. For $f(x,y) = 0$, the level curve is the x- and y-axes. For $f(x,y) \ne 0$, the level curves are hyperbolas with the x- and y-axes as asymptotes.
- C. The level curves are parabolas of the form $y = cx^2$.
- D. The level curves are circles centered at the origin with radii $r > 7$.

(d) Find the boundary of the function's domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The boundary is ☐. (Type an equation.)
- B. There are no boundary points.

(e) Determine if the domain is an open region, a closed region, or neither. Choose the correct answer below.

- A. The domain is neither open nor closed.
- B. The domain is closed.
- C. The domain is open.

(f) Decide if the domain is bounded or unbounded. Choose the correct answer below.

- A. The domain is unbounded.
- B. The domain is bounded.

ID: 13.1.29

20. Find all the second order partial derivatives of the given function.

$f(x,y) = x\ln(y-x)$

- A. $\frac{\partial^2 f}{\partial x^2} = \frac{x-2y}{(y-x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y-x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y-x)^2}$
- B. $\frac{\partial^2 f}{\partial x^2} = \frac{2y-x}{(y-x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y-x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y-x)^2}$
- C. $\frac{\partial^2 f}{\partial x^2} = \frac{x-2y}{(y-x)^2}; \frac{\partial^2 f}{\partial y^2} = \frac{x}{(y-x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y-x)^2}$
- D. $\frac{\partial^2 f}{\partial x^2} = \frac{x-2y}{(y-x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y-x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{y}{(y-x)^2}$

ID: 13.3-14