Main Examination period 2020 – January – Semester A

MTH4100/MTH4200: Calculus I

Duration: 2 hours

Student number ⬚⬚⬚⬚⬚⬚⬚⬚⬚   **Desk number** ⬚⬚⬚

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. Additional space is available at the end of the exam paper.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: O.F. Bandtlow and W. Huang

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Question	Mark	Comments
1	/ 8	
2	/ 8	
3	/ 8	
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6	/ 8	
7	/ 8	
8	/ 8	
9	/ 8	
10	/ 10	
11	/ 8	
12	/ 10	
Total		

Question 1. Find the natural domain and the corresponding range of the function

$$f(x) = \sqrt{\frac{x}{1-x}}.$$

[8]

Turn Over

Question 2. Sketch the function $f : [-2, 2] \to \mathbb{R}$ given by

$$f(x) = 1 - 2|\cos(\pi x)|.$$ [8]
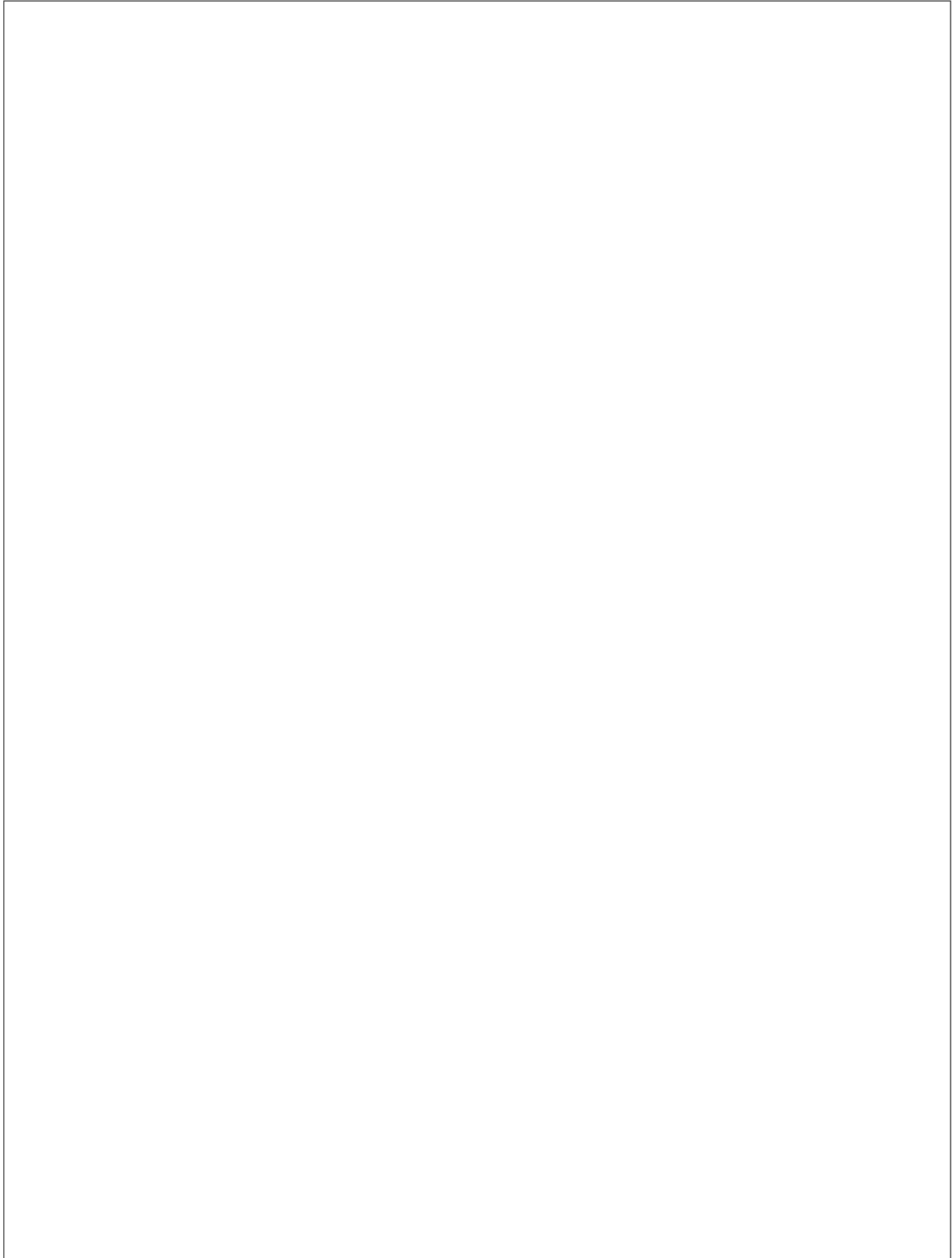
Question 3. Find the value of c such that the following function is continuous on $\mathbb{R}$

$$f(x) = \begin{cases} \dfrac{1 - e^{cx}}{x}, & x \neq 0; \\ 2, & x = 0. \end{cases}$$

[8]

Question 4.   Find all solutions of the equation

$$2\cos(x) + \mathrm{cosec}(x) = 0\,.$$

[8]

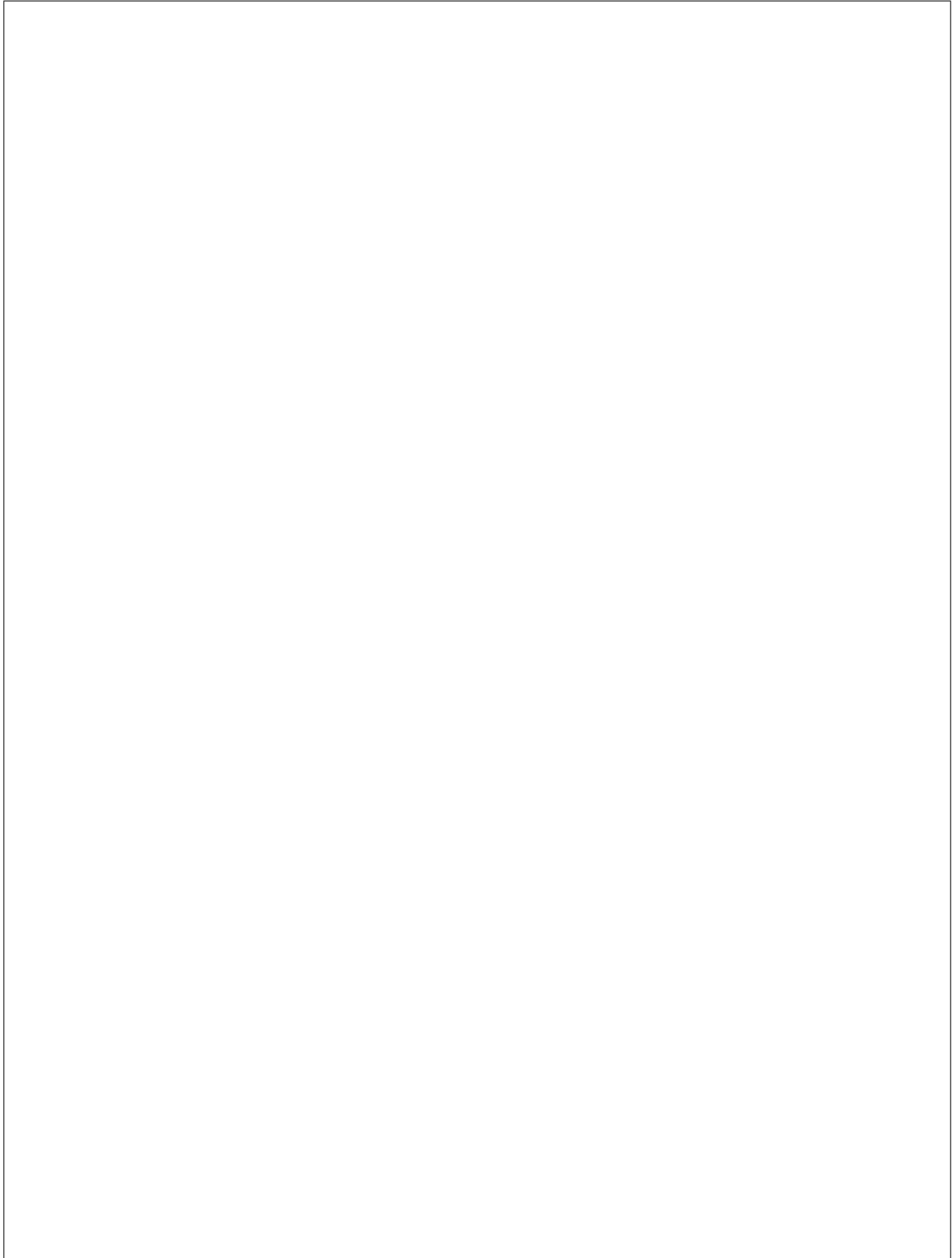**Question 5.** Find the derivative of the function

$$f(x) = \frac{\sin(\log(x))}{\cos(e^x)} \, .$$
[8]

Turn Over

Question 6. Find the equations of all vertical and all horizontal asymptotes of the function

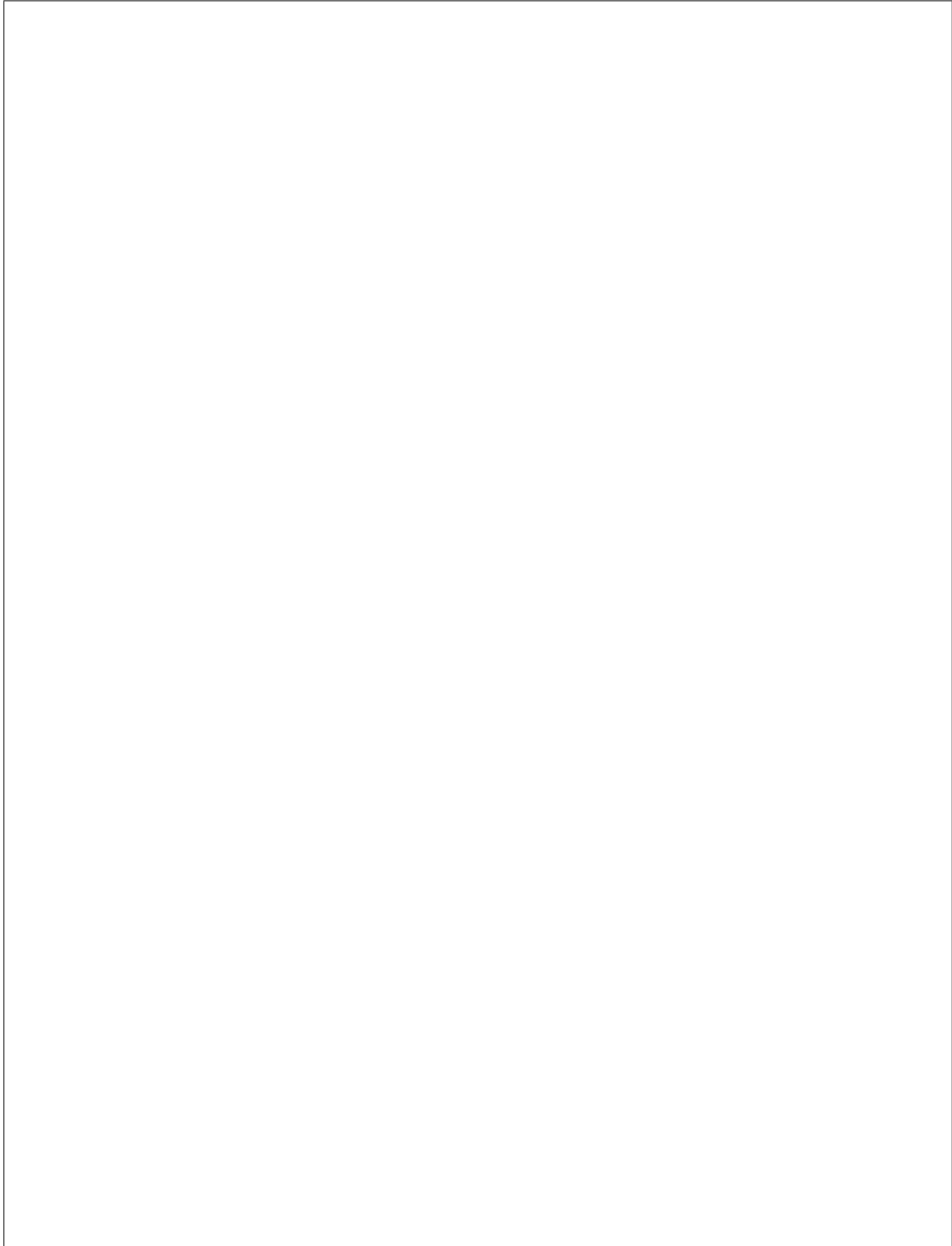$$f(x) = \frac{3x^2 - 9x + 6}{x^2 - x - 2}.$$

       [8]

Question 7.   Determine the linearisation of the following function at $-\frac{\pi}{2}$

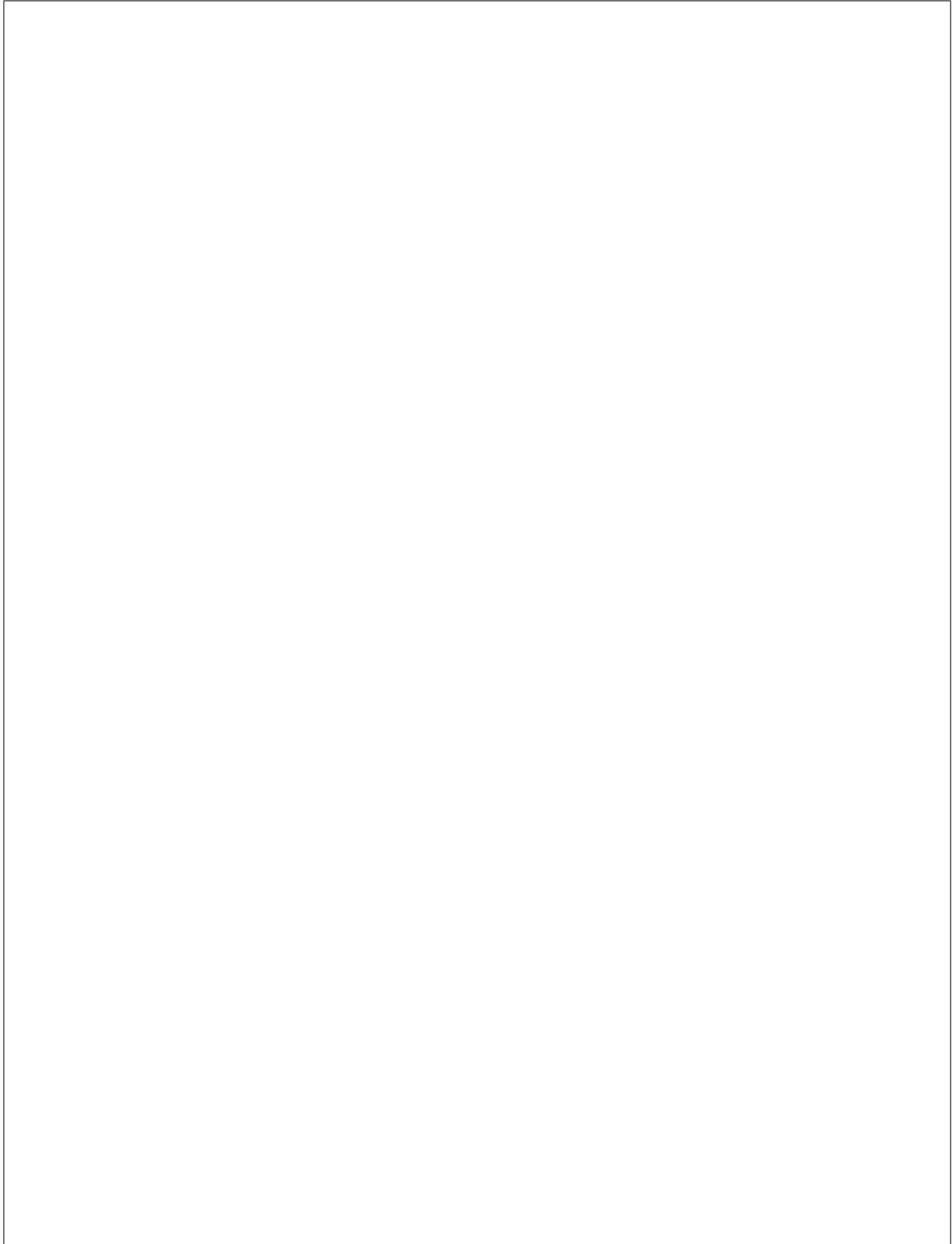$$f(x) = x^3 \cos(x).$$

[8]

Question 8.   Find the absolute maximum and the absolute minimum of the function

$$f(x) = x\sqrt{18 - x^2}$$

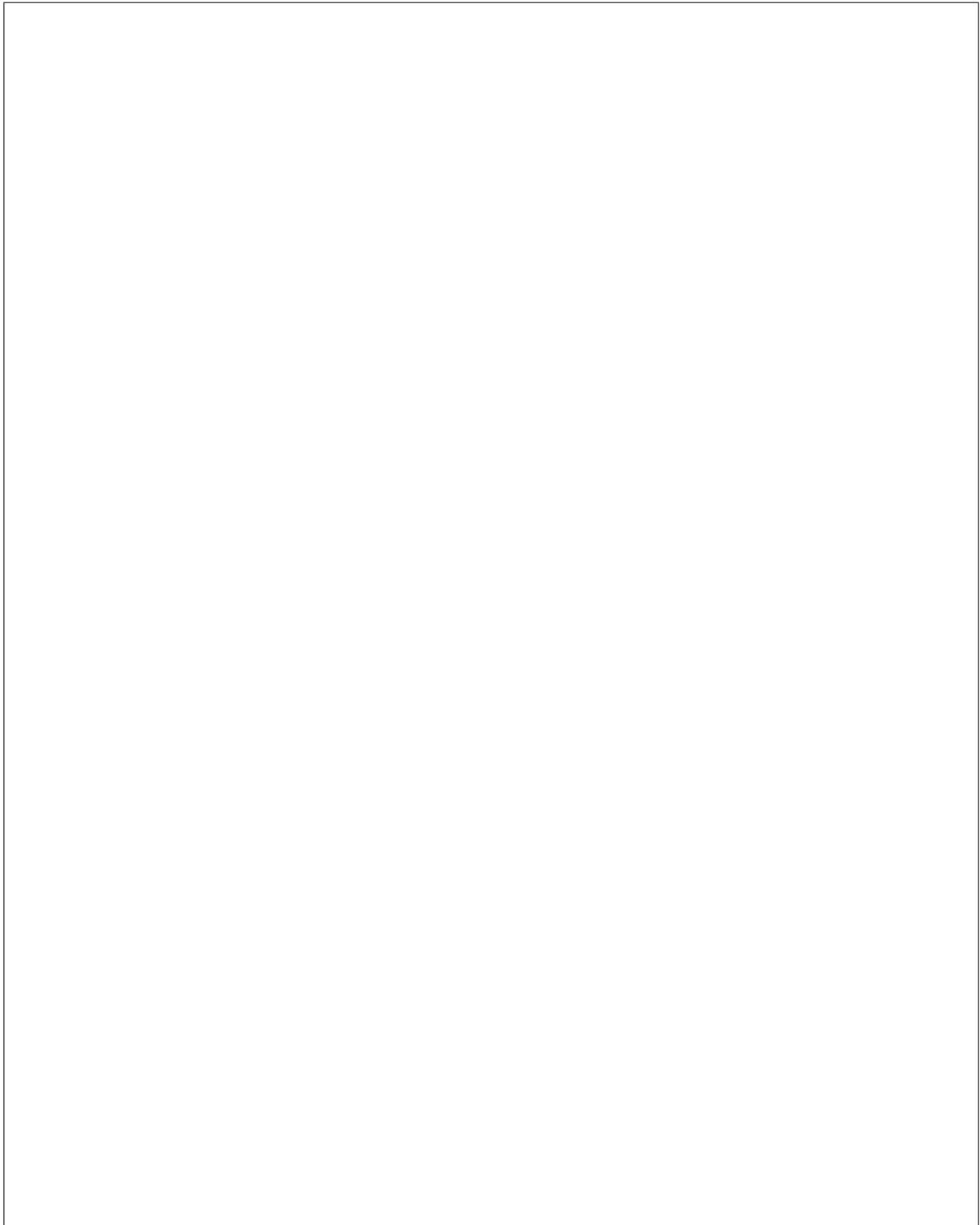on the interval $[-1, 4]$.   [8]

Question 9. Let
$$f(x) = 2x^5 + 5x^4 - 10x^3 + 2.$$

Find the open intervals on which f is increasing or decreasing, as well as the location of the local maxima and minima of f. [8]

$$f(x) = 2x^5 + 5x^4 - 10x^3 + 2.$$

Question 10. Show that the function

$$f(x) = 1 + x^2 - x^4 - \sin(\pi x)$$

has at least two points of inflexion in the interval $[-1, 1]$. **[10]**

Question 11. Find all antiderivatives of the function

$$f(x) = \frac{e^x}{1 + e^{2x}}.$$

[8]

Turn Over

Question 12. Evaluate the improper integral

$$\int_0^\infty x^2 e^{-x^3}\, dx.$$

[10]

End of Paper.

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