Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH4100: Calculus 1

Duration: 2 hours

Date and time: 1 May 2015, 10:00h–12:00h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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Examiner(s): Prof. B. Jackson

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Question 1. (a) Find the set of all $x \in \mathbb{R}$ which satisfy the inequality

$$|x^2 + 2x - 9| \leq 6.$$

[5 marks]

(b) Let $f(x) = 8x^3 + 1$. Determine the inverse function f^{-1}. **[5 marks]**

(c) Determine

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}\,.$$

[5 marks]

(d) Find the equation of the tangent to the curve $x^2 + y^3 = 2$ at the point $(1, 1)$. **[5 marks]**

(e) Find

$$\int \frac{2x}{x^2 + 2x + 1}\,\mathrm{d}x\,.$$

[5 marks]

(f) Find

$$\int_0^1 x^2\, e^x\,\mathrm{d}x\,.$$

[5 marks]

(g) Determine

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_1^{x^2} e^{\sin t}\,\mathrm{d}t\,.$$

[5 marks]

(h) Define the improper integral $\int_1^\infty x^{-2}\,\mathrm{d}x$ as a limit of definite integrals. Find this limit if it exists or give reasons if it does not exist. **[5 marks]**

Question 2. Consider the curve $y = f(x)$ for the function $f(x) = (1 - x^2)^{-1}$.

(a) Identify the domain of f and determine whether or not f is an even function or an odd function. **[2 marks]**

(b) Find $f'(x)$ and $f''(x)$. **[4 marks]**

(c) Find the critical points of f, determine where f is increasing or decreasing, and determine the behavior of f at each of its critical points. **[7 marks]**

(d) Determine the concavity of the curve and find any points of inflexion. **[5 marks]**

(e) Determine the behaviour of $f(x)$ as $x \to \pm\infty$ and identify any asymptotes. **[2 marks]**

(f) Plot key points, such as intercepts, critical points, and points of inflexion, and sketch the curve. **[5 marks]**

Question 3. Let f be a function defined on an open interval (a, b) and $c \in (a, b)$.

(a) Define the *derivative* of f at c and explain what it means to say that f *is differentiable on* (a, b). **[5 marks]**

(b) Explain what it means to say that f *has a local minimum at* c. What can you say about the derivative of f at c when f has a local minimum at c? Give a brief justification for your answer. **[5 marks]**

(c) Use your definition in (a) to prove the following statements.

(i)
$$\frac{\mathrm{d}}{\mathrm{d}x} x^{-2} = -2x^{-3}$$
[5 marks]

(ii)
$$\frac{\mathrm{d}}{\mathrm{d}x} f(x)^{-1} = -f(x)^{-2} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$$
whenever f is differentiable at x. **[5 marks]**

Question 4. (a) Define the *natural logarithm function* and state its domain and range. **[5 marks]**

(b) Prove the following statements, stating clearly any results which you use in your proofs.

(i) $\frac{\mathrm{d}}{\mathrm{d}x} \ln x = x^{-1}$. **[2 marks]**

(ii) $\ln x$ is an increasing function. **[2 marks]**

(iii) $\ln(x^q) = q \ln x$ for any rational number q. **[3 marks]**

(iv) $\lim_{x \to \infty} \ln x = \infty$. **[3 marks]**

End of Paper.