

**QUESTION 1**

Not yet answered Marked out of 20

Let A, B, C be distinct points in 3-space with position vectors  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  respectively. Suppose that A, B, C do not all lie on the same line, and let M be the plane passing through the three points A, B, C. Assume that M does not pass through the origin.

Let D be the point on the line segment AB such that  $|\vec{AD}| = \frac{1}{3}|\vec{AB}|$ . Let  $\mathbf{d}$  be the position vector of D.

$((\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})) \cdot (\mathbf{b} - \mathbf{c})$

Choose...

$2\mathbf{b}$

Choose...

$\mathbf{r} \cdot (\mathbf{b} - \mathbf{a}) = \mathbf{a} \cdot (\mathbf{b} - \mathbf{a})$

Choose...

$\frac{\mathbf{a} + 2\mathbf{b}}{3}$

Choose...

$\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$

Choose...

$(\mathbf{r} - \mathbf{a}) \cdot ((\mathbf{d} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})) = 0$

Choose...

$\frac{2\mathbf{a} + \mathbf{b}}{3}$

Choose...

$\frac{\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$

Choose...

$\frac{\mathbf{a} + \mathbf{c}}{2}$

Choose...

$\mathbf{a} \cdot \mathbf{a}$

Choose...

**QUESTION 2**

Not yet answered Marked out of 5

Let A be the point with coordinates (2, 0, 4) and let B be the point with coordinates (1, 0, 4). Let  $\mathbf{v}$  be the vector represented by the bound vector  $\vec{AB}$ . Let  $\Pi$  be a plane with the following properties:

- $\Pi$  contains the point with coordinates (-1, -2, 3)
- $\Pi$  is orthogonal to the vector  $\mathbf{v}$ .

What is the distance of the point with coordinates (-2, 0, -2) from  $\Pi$ ?

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 3**

Not yet answered Marked out of 5

Let  $l$  be the line given in parametric form by the equations

$x = 0 + 7\lambda$

$y = -1 + 7\lambda$

$z = 0 + 6\lambda$

What is the distance between the origin and the line  $l$ ?

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 4**

Not yet answered Marked out of 5

Let A be the point with coordinates (-4, 0, 2), let B be the point with coordinates (2, 6, 0), let C be the midpoint of the line segment AB, let D be the point with coordinates (1, 0, 0), and let E be the point with coordinates (0, 1, 0). Let  $\Pi$  be the plane passing through the three points C, D, E.

Suppose that the line  $2x - 6 = 2y - 6 = 2z - 6$  intersects the plane  $\Pi$  at the point P. Write down the x-coordinate of P.

[So, for example, if you find that P has coordinates (2.5, 4, 4) then you should enter 2.5 as your answer. Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer.]

Answer:

## QUESTION 5

Not yet answered Marked out of 5 Flag question

What is the distance between the line with Cartesian equation  $x-6 = y-7 = z-5$  and the line with Cartesian equation  $x-9 = y-10 = 3z-4$ ?

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between 6.53 and 7.03 will be marked as correct.]

Answer: 

## QUESTION 6

Not yet answered Marked out of 5 Flag question

Consider the following two matrices

$$A = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

- a. both A and B are in row echelon form
- b. A is in reduced row echelon form and B is in row echelon form
- c. both A and B are in reduced row echelon form
- d. A is in reduced row echelon form and B is not in row echelon form
- e. A is a 3 x 5 matrix

## QUESTION 7

Not yet answered Marked out of 5 Flag question

Consider the linear system

$$\begin{aligned} 3x_1 - 6x_2 + 3x_3 + 9x_4 &= 3 \\ 2x_1 - 3x_2 + 3x_3 + 4x_4 &= 4 \\ -3x_1 + 7x_2 - 2x_3 - 10x_4 &= -1 \end{aligned}$$

Bring the augmented matrix of the system to row echelon form, and state which of the variables are leading variables and which are free variables.

Select one:

- a.  $x_1$ ,  $x_2$  and  $x_4$  are the leading variables, while  $x_3$  is the free variable
- b.  $x_1$  and  $x_4$  are the leading variables, while  $x_2$  and  $x_3$  are the free variables
- c.  $x_1$ ,  $x_2$  and  $x_3$  are the leading variables, while  $x_4$  is the free variable
- d.  $x_1$  and  $x_2$  are the leading variables, while  $x_3$  and  $x_4$  are the free variables
- e.  $x_1$  and  $x_3$  are the leading variables, while  $x_2$  and  $x_4$  are the free variables

## QUESTION 8

Not yet answered Marked out of 5 Flag question

The system of linear equations:

$$\begin{aligned} 7x_1 + 5x_2 - 3x_3 &= 0 \\ 3x_1 - 2x_2 - x_3 &= -2 \end{aligned}$$

- a. is a 2 x 3 overdetermined inconsistent system
- b. is a 2 x 3 overdetermined consistent system
- c. is a 3 x 2 underdetermined inconsistent system
- d. is a 3 x 2 underdetermined consistent system
- e. None of the others

## QUESTION 9

Not yet answered Marked out of 5 Flag question

If  $x$ ,  $y$  and  $z$  satisfy the system of equations below:

$$\begin{aligned} 5x - y + 2z &= -3 \\ -5x + 2y - 2z &= 1 \\ 5x - y + 5z &= 3 \end{aligned}$$

write the value of  $15x + y^2 + 3z$

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 10**

Not yet answered Marked out of 5 Flag question

Consider the following two matrices:

$$A = \begin{pmatrix} -1 & 4 \\ 2 & -1 \\ 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

Which of the following four products exist?

$$A^2, B^3, AB, BA$$

Select one:

- a.  $B^3$  and  $BA$  exist, while  $A^2$  and  $AB$  do not exist
- b.  $B^3, AB$  and  $BA$  exist, while  $A^2$  does not exist
- c. only  $B^3$  exists
- d.  $A^2, B^3$  and  $BA$  exist, while  $AB$  does not exist
- e. none of the four products exist

**QUESTION 11**

Not yet answered Marked out of 5 Flag question

Consider the matrix  $A = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$  and the vector  $\mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$ . Let  $B = (A - A^T)^2$  and define vector  $\mathbf{v} = B\mathbf{u}$ . If  $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ , write down the value of  $v_1$ .

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 12**

Not yet answered Marked out of 5 Flag question

Consider matrix  $A = \begin{pmatrix} 2 & 0 & 0 & 2 \\ 3 & 0 & 1 & -2 \\ -7 & 2 & 4 & 1 \\ 0 & 0 & 0 & 5 \end{pmatrix}$ .

Calculate  $\det(A^3)$

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 13**

Not yet answered Marked out of 5 Flag question

Consider matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ -1 & 0 & -2 \end{pmatrix}$$

and vectors  $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  and  $\mathbf{y} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ .

Let  $\mathbf{w} = A^{-1}\mathbf{x}$ . Evaluate the scalar product of  $\mathbf{y}$  and  $\mathbf{w}$ . (Hint: Use the Gauss-Jordan algorithm to find the inverse  $A^{-1}$  of the matrix  $A$  first)

[Please enter your answer numerically in decimal format. You will be marked correct as long as what you enter is within 0.25 of the correct answer. So for example, if the correct answer is 6.78 then any input that lies between between 6.53 and 7.03 will be marked as correct.]

Answer:

**QUESTION 14**

Not yet answered Marked out of 20 Flag question

**Note: For this question, submissions must be handwritten and converted into a single file before uploading! PDF format is preferred but image files are also allowed.**

a) **(4 marks)** Each of the following equations defines a geometric object in 3-space. Describe in words what the object is for each of the equations. Be as specific as possible.

(i)  $x^2 + xy + xz = 0$

(ii)  $x = y - 1 = z - 2$

b) **(4 marks)** Let  $l_1$  and  $l_2$  be lines in 3 space such that

- $l_1$  passes through the point with coordinates (3, 4, 5) in the direction of  $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$
- $l_2$  passes through the points with coordinates (8, 0, 1) and (8, 0, -1)

Write down the equations of  $l_1$  and  $l_2$  in **parametric** as well as **Cartesian** forms.

c) **(2 marks)** Let  $l_1$  and  $l_2$  be as above. Describe precisely the intersection  $l_1 \cap l_2$  and justify your answer.

For the next three parts of this question, consider the linear system

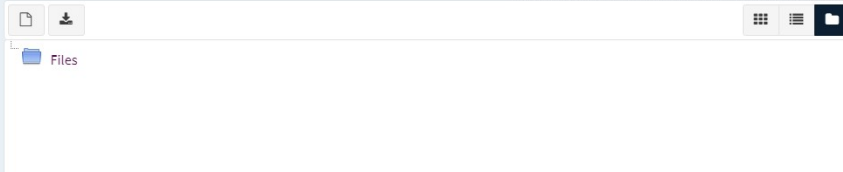
$$\begin{array}{r} x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ -x_1 - 2x_2 - x_4 = 2 \\ 2x_1 + 4x_2 + x_3 + 4x_4 = 3 \end{array}$$

d) **(1 marks)** Write down the augmented matrix of the system.

e) **(5 marks)** Bring the augmented matrix to row echelon form, indicating which elementary row operation you have used at each step.

f) **(4 marks)** State which of the variables are leading variables and which are free variables of the reduced system, and write down the solution set of the system.

Maximum file size: Unlimited, maximum number of files: 1



Accepted file types

Image files .ai .bmp .gdraw .gif .ico .jpe .jpeg .jpg .pct .pic .pict .png .svg .svgz .tif .tiff  
PDF document .pdf

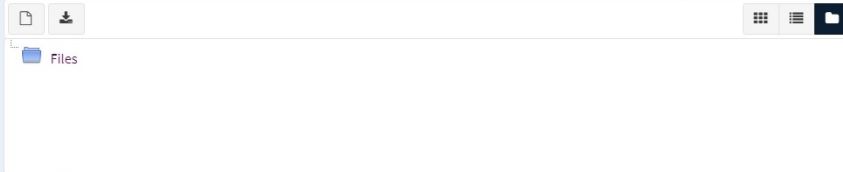
**QUESTION 15**

Not yet answered Not graded Flag question

You are invited here to upload scans of your rough work for this exam; in the event that you fail the assessment, we will use these to see whether there are method marks we can award to help you pass.

Your submissions must be handwritten and converted into a single PDF or image file.

Maximum file size: Unlimited, maximum number of files: 1



Accepted file types

Image files .ai .bmp .gdraw .gif .ico .jpe .jpeg .jpg .pct .pic .pict .png .svg .svgz .tif .tiff  
PDF document .pdf