MTH4113 / MTH4213 - NUMBERS, SETS AND FUNCTIONS - 2021/22

SEMESTER A FINAL ASSESSMENT

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In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

Please read the following message

I understand that it is important that the attempt I am about to make is all my own work. I understand what constitutes plagiarism or cheating, and I will not undertake such activities.

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QUESTION 1

Not yet answered Marked out of 12

Suppose P, Q and R are statements. Complete the truth table for the statement (P or Q) $\Rightarrow$ (P and (not R)).

P	Q	R	(P or Q) $\Rightarrow$ (P and (not R))
true	true	true	
true	true	false	
true	false	true	
true	false	false	
false	true	true	
false	true	false	
false	false	true	
false	false	false	

QUESTION 2

Not yet answered Marked out of 10

Suppose A, B and C are finite sets of real numbers.

Decide whether each of the following is a set, a number, a statement or meaningless.

$|A| + |C|$

$A \cup (B \cap C)$

$B = \{n \in \mathbb{N} : n + 1 \in A\}$

$A \cap B$ is disjoint

$((A \cup B) \cup C) \cup A$

QUESTION 3

Not yet answered Marked out of 10

True or false?

25 is a factor of 5.

$4 \mid 0$.

If $a, b, c \in \mathbb{Z}$ and $a \mid b$ and $b \mid c$, then $c \mid a$.

If $a \in \mathbb{Z}$ and $6 \nmid a$, then $3 \nmid a$.

If $a, b \in \mathbb{N}$, then $\mathrm{lcm}(a^2, b^2) = \mathrm{lcm}(a, b)^2$.

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QUESTION 4

Not yet answered Marked out of 15

Let A and B be finite sets satisfying $|A| = 6$, $|B| = 7$ and $|A \cap B| = 2$.

Calculate the following:

$\binom{|A|}{3} =$ ____

$|A \cup B| =$ ____

The number of non-empty subsets of B is ____

$\left|\{S : S \subseteq A,\ |S| = 3 \text{ and } S \cap B = \emptyset\}\right| =$ ____

$\max\left\{|S| : S \subseteq A \cup B,\ |S \cap A| \text{ is even and } |S \cap B| \text{ is even}\right\} =$ ____

QUESTION 5

Not yet answered Marked out of 15

Decide whether each of the following functions is injective and/or surjective.

$f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = 2n$.

- ○ f is injective but not surjective
- ○ f is surjective but not injective
- ○ f is both injective and surjective
- ○ f is neither injective nor surjective.

$g : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(m, n) = (m^2, m + n)$.

- ○ g is injective but not surjective
- ○ g is surjective but not injective
- ○ g is both injective and surjective
- ○ g is neither injective nor surjective.

$h : \mathbb{N} \to \mathbb{N}$ defined by

$h(n) =$ the number of digits of n^2.

- ○ h is injective but not surjective
- ○ h is surjective but not injective
- ○ h is both injective and surjective
- ○ h is neither injective nor surjective.

QUESTION 6

Not yet answered Marked out of 15

Find the supremum of each of the following sets. (If the supremum is infinite, enter the word "infinity". If it is a real number, round it to 1 decimal place.)

$\{4, 1, 2, 3\}$

$\{(x-1)^2 \ : \ x \in \mathbb{R},\ 0 \leqslant x < 2\}$

$\left\{\frac{n^2}{4} \ : \ n \in \mathbb{Z}\right\}$

$\left\{\frac{n+1}{n} \ : \ n \in \mathbb{N}\right\}$

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QUESTION 7

Not yet answered Marked out of 8

Use Euclid's algorithm to find the greatest common divisor of 52 and 80. Hence find the lowest common multiple of 52 and 80.

Upload a PDF of your solution. Show your working clearly as well as giving the answer.

Maximum file size: Unlimited, maximum number of files: 1

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QUESTION 8

Not yet answered Marked out of 10

Give a proof by induction of the following statement.

If n is a positive integer, then

$$\sum_{a=1}^{n}(3a+2) = \frac{n(3n+7)}{2}.$$

(Upload your written proof in PDF format.)

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Accepted file types

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QUESTION 9

Not yet answered Marked out of 5

Here's an incorrect theorem with an incorrect proof.

Theorem: There is no rational number x such that $1 < x < 2$.

Proof: We use proof by contradiction. So suppose there are rational numbers between 1 and 2, and let x be the smallest one. Because x is rational, we can write $x = \frac{a}{b}$ where a and b are integers. Now let $y = \frac{x+1}{2}$. Then $y = \frac{a+b}{2b}$, so y is rational.

The assumption that $x > 1$ gives $x + 1 > 2$, which implies that $y > 1$. The assumption that $x > 1$ also gives $2x > x + 1$, which implies that $y < x$.

But now y is a rational number between 1 and 2 which is smaller than x. This contradicts the assumption that x was the smallest one. So the theorem is true using proof by comntradiction. □.

Explain in a few sentences what's wrong with the proof. (You can either type your answer in the box or upload a scan of your written answer as a PDF.)

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