Queen Mary
University of London

Main Examination period 2019

MTH4113, MTH4213: Numbers, Sets and Functions

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: R. Johnson and B. Jackson

In this exam $\mathbb{N} = \{1, 2, 3, 4, \dots\}$.

Question 1. Let

$$A = \{1, 3, 4\}; \quad B = \{1, 2, 5\}; \quad C = \{1, 3, 5\}.$$

For each of the following expressions, state whether it is a set, a number, or a statement. For those expressions which are statements, state whether they are true or false (giving a reason). For those expressions which are sets or numbers, evaluate them (showing your working).

(a) $A \cup B$ **[4]**

(b) $A \subseteq (B \cup C)$ **[4]**

(c) $|A| \in A$ **[4]**

(d) $|A \cap B| / |A \cup B|$ **[4]**

(e) $|A \cup B| = |A| + |C| - |A \cap B|$ **[4]**

Question 2.
Suppose that $P(x)$ and $Q(x)$ are mathematical statements about some object x, and X is some set.

(a) Write down a roadmap (as in lectures, I mean by this an outline of the structure a proof could have including the starting point and conclusion but omitting the details) for proving the following statement is true. **[4]**

For all $x \in X$, $P(x) \Rightarrow Q(x)$.

Let S be the statement:

For all $x, y \in \mathbb{R}$, if $x + y = 0$ then $xy \leq 0$.

(b) Decide whether the statement S is true or false, giving a proof or counterexample as appropriate. **[6]**

(c) Write down the statement obtained by replacing the implication in S by its converse. Decide whether this new statement is true or false, giving a proof or counterexample as appropriate. **[5]**

(d) Write down the statement obtained by replacing the implication in S by its contrapositive. Decide whether this new statement is true or false, giving a proof or counterexample as appropriate. **[5]**

Question 3. Let $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ be the function defined by

$$f(n) = \{n, n+1, \ldots, 2n\}.$$

(a) Find $f(2)$ and $f(3)$. **[4]**

(b) Decide whether each of the following statements is true or false giving a brief reason for each answer.

(i) For all $n \in \mathbb{N}$, we have $|f(n)| = n + 1$. **[4]**

(ii) For all $i, j \in \mathbb{N}$, we have $f(i) \cap f(j) \neq \emptyset$. **[4]**

(iii) The range of f is a finite subset of $\mathbb{N}$. **[4]**

(iv) Every element of the range of f is a finite subset of $\mathbb{N}$. **[4]**

Question 4.

(a) Explain what is meant by the **complex plane** and how to represent a complex number $a + b\mathrm{i}$ on it. **[5]**

(b) Let z be the complex number $7 - 3\mathrm{i}$. Find:

(i) z^2 **[3]**

(ii) $|z|$ **[3]**

(iii) The complex number corresponding to the image of z under reflection in the real axis of the complex plane. **[3]**

(iv) A complex number y such that $z + y$ is a negative real number. **[3]**

(v) A complex number w such that zw is a negative real number. **[3]**

Question 5.

(a) Define what it means for a to **divide** n (written $a \mid n$) where a and n are integers. **[4]**

(b) Prove that for all integers a and n, if $a \mid n$ then $a^2 \mid n^2$. **[4]**

(c) Identify the mistake in the following false proof that 7 divides $2^{3n} - 1$ for all $n \in \mathbb{N}$. **[4]**

> We have that $2^{3n} = 8^n$ and so 2^{3n} is a multiple of 8. It follows that $2^{3n} - 1$ is a multiple of $8 - 1$. Hence $2^{3n} - 1$ is a multiple of 7.

(d) Use induction to give a correct proof that 7 divides $2^{3n} - 1$ for all $n \in \mathbb{N}$. **[8]**

End of Paper.