Queen Mary
University of London

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, **you must submit within the first 3 hours of the assessment period**.

Examiners: A. Baule, A. Shestopaloff

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Question 1 [25 marks].

Please access the questions on QMplus under "Semester A Final Assessment 2020/21". [**25**]

Question 2 [19 marks]. A new index-linked gilt is issued on 1.1.2019. The gilt has a coupon rate of 0.25% of nominal paid twice yearly in arrears and the bond will mature in 25 years time at 110% of indexed nominal. There is a no time lag in indexing payments. The bond is indexed by reference to the Retail Prices Index and the relevant values are shown below.

Year	Jan	April	July	Oct
2018	276.0	279.7	281.7	284.5
2019	283.0	288.2	289.5	290.4
2020	290.6	292.6	294.2	-

An investor purchases £1000 nominal of the gilt at issue and sells one year later after receiving two coupon payments.

(a) Calculate the coupon payments that the investor receives. [**6**]

(b) Calculate the sale price of the bond on 1.1.2020 if the real rate of return is 1.5% per annum. [**6**]

(c) Calculate the investor's yield assuming he bought the bond for £850.55 on 1.1.2019. [**7**]

Question 3 [16 marks].

An investor is considering buying a property with a 92 year lease. The property is tenanted with a current rental income of £4,500 per month payable monthly in advance. Expenses for management and maintenance are £13,000 per year payable quarterly in arrears.

(a) Determine the value of the property assuming an effective interest rate of 2.5% per annum. Assume that neither rent nor expenses increase. [**6**]

(b) Assume now that rents are reviewed every 3 years and the next review is in 2 years time. Determine the maximum price that the investor should offer to achieve a pre-tax return of 5% per annum, assuming that the rate of future rental increase will be 2% per annum and that expenses do not increase. [**10**]

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Question 4 [20 marks].

(a) Use mortality given by table AMC00 ultimate values. You can find the table in the appendix.

(i) Explain in words the meaning of the symbol ${}_{2|4}q_{35}$ and calculate its value. **[5]**

(ii) Calculate the value of the symbol ${}_{3.5}p_{22}$. State any assumptions made. **[5]**

(b) Consider the survival function

$$s(x) = \frac{16 - x^2}{16}, \quad \text{for} \quad 0 \leqslant x \leqslant 4.$$

(i) Show that $s(x)$ is a valid survival function. **[6]**

(ii) Calculate the complete expectation of life at age $x = 2$ using this survival function. **[4]**

Question 5 [20 marks].

(a) Dan Brown, aged 55, takes out a whole-life assurance policy with a sum insured of £100,000 paid at end of year of death. Assume an effective interest rate of 4% per annum and mortality given by the AMC00 Select Table.

(i) Show that the cost of the life assurance policy as a net single premium is approximately £36,297. **[4]**

(ii) If he pays for the assurance annually in advance for life, what is the annual premium? **[4]**

(iii) If he instead pays for the assurance by making quarterly payments for life in arrears, what is the quarterly premium? **[4]**

(b) Consider a life assurance policy for a life of age x, in which a £1 death benefit is paid at the instant of death, but only if death occurs within n years from the start of the contract. Determine an expression for the expected present value of such a policy assuming that the complete further lifetime $T(x)$ is uniformly distributed on the interval $(0, \omega - x)$, where $\omega > 0$ is a constant, and $0 < x < \omega$. State any assumptions made. **[8]**

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AMC00 Male Life Table

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
17	9997.5094		10000.0000	17
18	9992.9305	9994.6901	9995.4200	18
19	9988.3337	9990.1025	9990.8321	19
20	9983.6991	9985.4871	9986.2163	20
21	9979.0567	9980.8438	9981.5827	21
22	9974.3666	9976.1827	9976.9213	22
23	9969.6487	9971.4740	9972.2222	23
24	9964.8832	9966.7276	9967.4854	24
25	9960.0701	9961.9336	9962.7010	25
26	9955.1996	9957.0921	9957.8691	26
27	9950.2618	9952.1832	9952.9698	27
28	9945.2469	9947.1972	9947.9933	28
29	9940.1847	9942.1340	9942.9398	29
30	9935.0655	9936.9939	9937.7794	30
31	9929.8794	9931.7671	9932.5024	31
32	9924.5767	9926.4238	9927.0892	32
33	9919.1182	9920.9344	9921.5201	33
34	9913.4841	9915.2596	9915.7755	34
35	9907.6152	9909.3799	9909.8162	35
36	9901.4922	9903.2460	9903.6126	36
37	9895.0760	9896.8385	9897.1357	37
38	9888.3275	9890.1087	9890.3363	38
39	9881.1880	9882.9977	9883.1559	39
40	9873.5893	9875.4570	9875.5558	40
41	9865.4534	9867.4085	9867.4578	41
42	9856.7223	9858.7745	9858.7942	42
43	9847.3090	9849.4875	9849.4875	43
44	9837.1169	9839.4312	9839.4312	44
45	9826.0499	9828.5291	9828.5291	45
46	9813.9735	9816.6562	9816.6562	46
47	9800.7439	9803.6786	9803.6786	47
48	9786.2090	9789.4437	9789.4437	48
49	9770.1885	9773.7708	9773.7708	49
50	9752.4550	9756.4712	9756.4712	50
51	9732.8031	9737.3192	9737.3192	51
52	9710.9228	9716.0627	9716.0627	52
53	9686.5083	9692.4138	9692.4332	53
54	9659.2100	9666.0213	9666.1182	54
55	9628.6170	9636.5205	9636.7719	55
56	9594.3554	9603.4960	9604.0069	56
57	9555.9547	9566.5317	9567.3964	57
58	9512.9284	9525.1558	9526.4767	58

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
59	9464.7096	9478.8531	9480.7305	59
60	9410.6849	9427.0212	9429.5820	60
61	9350.1805	9369.0144	9372.4010	61
62	9282.4873	9304.1589	9308.5187	62
63	9206.8320	9231.7028	9237.1968	63
64	9122.3956	9150.8360	9157.6369	64
65	9028.3257	9060.7100	9069.0001	65
66	8923.7129	8960.4327	8970.3747	66
67	8807.6361	8849.0571	8860.8106	67
68	8679.1539	8725.6105	8739.3111	68
69	8537.3365	8589.1250	8604.8568	69
70	8381.2654	8438.6022	8456.4058	70
71	8210.1056	8273.0800	8292.9181	71
72	8023.0608	8091.6666	8113.3848	72
73	7819.4936	7893.5124	7916.8461	73
74	7598.8879	7677.9138	7702.4263	74
75	7360.9680	7444.3113	7469.3894	75
76	7105.6585	7192.3651	7217.1705	76
77	6833.2053	6921.9772	6945.4296	77
78	6543.2750	6633.3614	6654.1244	78
79	6234.6920	6326.2215	6343.5631	79
80	5905.8869	6000.3050	6014.4463	80
81	5557.1270	5655.4241	5667.9541	81
82	5191.0860	5293.5691	5305.7888	82
83	4811.5773	4917.5469	4930.2080	83
84	4422.4381	4531.0383	4544.0494	84
85	4027.2597	4137.4915	4150.7347	85
86	3630.1730	3740.8772	3754.2191	86
87	3235.7358	3345.6473	3358.9374	87
88	2848.8406	2956.5986	2969.6769	88
89	2474.5254	2578.7306	2591.4291	89
90	2117.7709	2217.0436	2229.1940	90
91		1876.3069	1887.7528	91
92			1571.4202	92
93			1283.8047	93
94			1027.5920	94
95			804.3661	95
96			614.5164	96
97			457.2229	97
98			330.5502	98
99			231.6268	99
100			156.9026	100

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AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17
18	4932.79	4933.66	4934.02	18
19	4740.89	4741.73	4742.07	19
20	4556.43	4557.25	4557.58	20
21	4379.15	4379.93	4380.25	21
22	4208.74	4209.50	4209.82	22
23	4044.95	4045.69	4045.99	23
24	3887.51	3888.23	3888.53	24
25	3736.19	3736.89	3737.18	25
26	3590.73	3591.42	3591.70	26
27	3450.92	3451.58	3451.85	27
28	3316.52	3317.17	3317.43	28
29	3187.33	3187.96	3188.22	29
30	3063.17	3063.76	3064.00	30
31	2943.81	2944.37	2944.59	31
32	2829.08	2829.61	2829.80	32
33	2718.77	2719.27	2719.43	33
34	2612.72	2613.19	2613.32	34
35	2510.74	2511.19	2511.30	35
36	2412.68	2413.11	2413.20	36
37	2318.39	2318.80	2318.87	37
38	2227.70	2228.10	2228.15	38
39	2140.47	2140.86	2140.90	39
40	2056.56	2056.95	2056.97	40
41	1975.83	1976.22	1976.23	41
42	1898.16	1898.55	1898.56	42
43	1823.41	1823.81	1823.81	43
44	1751.46	1751.87	1751.87	44
45	1682.20	1682.63	1682.63	45
46	1615.52	1615.96	1615.96	46
47	1551.29	1551.75	1551.75	47
48	1489.41	1489.90	1489.90	48
49	1429.78	1430.30	1430.30	49
50	1372.29	1372.86	1372.86	50
51	1316.85	1317.47	1317.47	51
52	1263.36	1264.03	1264.03	52
53	1211.71	1212.45	1212.46	53
54	1161.83	1162.65	1162.66	54
55	1113.60	1114.52	1114.55	55
56	1066.96	1067.98	1068.03	56
57	1021.82	1022.95	1023.04	57
58	978.09	979.35	979.49	58

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
59	935.71	937.11	937.29	59
60	894.58	896.14	896.38	60
61	854.65	856.37	856.68	61
62	815.83	817.73	818.11	62
63	778.05	780.16	780.62	63
64	741.27	743.58	744.13	64
65	705.41	707.94	708.59	65
66	670.42	673.18	673.92	66
67	636.25	639.24	640.09	67
68	602.85	606.08	607.03	68
69	570.19	573.65	574.70	69
70	538.24	541.92	543.07	70
71	506.97	510.86	512.08	71
72	476.36	480.44	481.73	72
73	446.42	450.65	451.98	73
74	417.14	421.48	422.82	74
75	388.54	392.94	394.26	75
76	360.64	365.04	366.30	76
77	333.47	337.80	338.95	77
78	307.04	311.27	312.24	78
79	281.31	285.44	286.22	79
80	256.22	260.32	260.93	80
81	231.82	235.92	236.44	81
82	208.22	212.33	212.82	82
83	185.58	189.66	190.15	83
84	164.01	168.03	168.52	84
85	143.61	147.54	148.01	85
86	124.47	128.26	128.72	86
87	106.68	110.30	110.74	87
88	90.31	93.73	94.14	88
89	75.43	78.60	78.99	89
90	62.07	64.98	65.34	90
91		52.88	53.20	91
92			42.58	92
93			33.45	93
94			25.74	94
95			19.38	95
96			14.23	96
97			10.18	97
98			7.08	98
99			4.77	99
100			3.11	100

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AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17
18	115879.55	115880.77	115881.13	18
19	110945.59	110946.76	110947.11	19
20	106203.56	106204.70	106205.03	20
21	101646.03	101647.13	101647.45	21
22	97265.82	97266.89	97267.20	22
23	93056.05	93057.08	93057.39	23
24	89010.09	89011.10	89011.39	24
25	85121.59	85122.57	85122.86	25
26	81384.45	81385.41	81385.69	26
27	77792.78	77793.72	77793.99	27
28	74340.96	74341.87	74342.13	28
29	71023.58	71024.44	71024.70	29
30	67835.43	67836.24	67836.49	30
31	64771.51	64772.26	64772.48	31
32	61827.01	61827.70	61827.89	32
33	58997.30	58997.93	58998.09	33
34	56277.95	56278.53	56278.66	34
35	53664.69	53665.23	53665.34	35
36	51153.45	51153.95	51154.04	36
37	48740.30	48740.77	48740.84	37
38	46421.48	46421.92	46421.97	38
39	44193.38	44193.79	44193.82	39
40	42052.51	42052.91	42052.93	40
41	39995.55	39995.95	39995.96	41
42	38019.32	38019.72	38019.72	42
43	36120.76	36121.17	36121.17	43
44	34296.94	34297.35	34297.35	44
45	32545.06	32545.48	32545.48	45
46	30862.41	30862.85	30862.85	46
47	29246.43	29246.89	29246.89	47
48	27694.65	27695.14	27695.14	48
49	26204.72	26205.24	26205.24	49
50	24774.37	24774.94	24774.94	50
51	23401.47	23402.08	23402.08	51
52	22083.94	22084.61	22084.61	52
53	20819.83	20820.58	20820.58	53
54	19607.27	19608.12	19608.13	54
55	18444.47	18445.44	18445.47	55
56	17329.76	17330.87	17330.93	56
57	16261.53	16262.80	16262.89	57
58	15238.27	15239.71	15239.85	58

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
59	14258.53	14260.18	14260.36	59
60	13320.96	13322.83	13323.07	60
61	12424.28	12426.38	12426.69	61
62	11567.26	11569.63	11570.01	62
63	10748.78	10751.44	10751.90	63
64	9967.77	9970.73	9971.28	64
65	9223.22	9226.50	9227.15	65
66	8514.21	8517.82	8518.56	66
67	7839.85	7843.79	7844.64	67
68	7199.32	7203.60	7204.55	68
69	6591.87	6596.47	6597.52	69
70	6016.77	6021.68	6022.82	70
71	5473.35	5478.53	5479.75	71
72	4960.98	4966.38	4967.67	72
73	4479.04	4484.61	4485.94	73
74	4026.96	4032.62	4033.96	74
75	3604.16	3609.81	3611.14	75
76	3210.07	3215.62	3216.88	76
77	2844.13	2849.44	2850.58	77
78	2505.65	2510.66	2511.63	78
79	2193.87	2198.61	2199.39	79
80	1907.94	1912.56	1913.17	80
81	1647.13	1651.72	1652.24	81
82	1410.71	1415.31	1415.80	82
83	1197.92	1202.49	1202.97	83
84	1007.84	1012.34	1012.82	84
85	839.45	843.83	844.31	85
86	691.61	695.84	696.30	86
87	563.10	567.14	567.58	87
88	452.62	456.42	456.84	88
89	358.78	362.31	362.70	89
90	280.12	283.35	283.71	90
91		218.05	218.37	91
92			165.17	92
93			122.59	93
94			89.14	94
95			63.39	95
96			44.02	96
97			29.78	97
98			19.60	98
99			12.52	99
100			7.75	100

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AMC00 Male Life Functions (4%)

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
17	478.10		479.31	17
18	475.89	476.71	477.05	18
19	473.75	474.54	474.88	19
20	471.68	472.45	472.77	20
21	469.68	470.42	470.74	21
22	467.74	468.47	468.77	22
23	465.87	466.57	466.86	23
24	464.05	464.73	465.02	24
25	462.28	462.94	463.22	25
26	460.56	461.21	461.48	26
27	458.89	459.52	459.78	27
28	457.25	457.86	458.12	28
29	455.66	456.25	456.50	29
30	454.11	454.67	454.91	30
31	452.60	453.13	453.34	31
32	451.12	451.62	451.80	32
33	449.65	450.12	450.27	33
34	448.18	448.63	448.76	34
35	446.72	447.14	447.25	35
36	445.24	445.65	445.74	36
37	443.76	444.15	444.22	37
38	442.25	442.64	442.69	38
39	440.72	441.10	441.13	39
40	439.16	439.53	439.55	40
41	437.54	437.92	437.93	41
42	435.88	436.26	436.26	42
43	434.15	434.54	434.54	43
44	432.35	432.75	432.75	44
45	430.47	430.88	430.88	45
46	428.50	428.92	428.92	46
47	426.42	426.87	426.87	47
48	424.23	424.70	424.70	48
49	421.91	422.41	422.41	49
50	419.43	419.98	419.98	50
51	416.80	417.39	417.39	51
52	413.98	414.62	414.62	52
53	410.95	411.66	411.66	53
54	407.70	408.49	408.50	54
55	404.20	405.08	405.10	55
56	400.43	401.41	401.46	56
57	396.37	397.46	397.55	57
58	392.01	393.21	393.34	58

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
59	387.30	388.64	388.82	59
60	382.24	383.72	383.95	60
61	376.79	378.43	378.73	61
62	370.93	372.74	373.11	62
63	364.64	366.64	367.09	63
64	357.89	360.09	360.62	64
65	350.67	353.07	353.70	65
66	342.95	345.57	346.29	66
67	334.71	337.55	338.37	67
68	325.95	329.02	329.93	68
69	316.66	319.94	320.95	69
70	306.83	310.32	311.42	70
71	296.46	300.15	301.32	71
72	285.56	289.42	290.66	72
73	274.15	278.16	279.44	73
74	262.26	266.38	267.67	74
75	249.92	254.10	255.37	75
76	237.17	241.36	242.57	76
77	224.08	228.21	229.31	77
78	210.67	214.70	215.64	78
79	196.93	200.88	201.63	79
80	182.84	186.76	187.35	80
81	168.47	172.39	172.90	81
82	153.96	157.90	158.37	82
83	139.50	143.41	143.88	83
84	125.24	129.10	129.56	84
85	111.32	115.08	115.54	85
86	97.87	101.50	101.94	86
87	85.02	88.49	88.91	87
88	72.90	76.17	76.57	88
89	61.63	64.67	65.04	89
90	51.30	54.08	54.42	90
91		44.49	44.80	91
92			36.23	92
93			28.74	93
94			22.32	94
95			16.94	95
96			12.54	96
97			9.04	97
98			6.33	98
99			4.29	99
100			2.81	100

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AMC00 Male Life Functions (4%)

Age x	$A_{[x]}$	A_x	$\ddot{a}_{[x]}$	$\ddot{a}_x$	Age x
17	0.09315	0.09337	23.578	23.572	17
18	0.09647	0.09669	23.492	23.486	18
19	0.09993	0.10014	23.402	23.396	19
20	0.10352	0.10373	23.309	23.303	20
21	0.10725	0.10747	23.211	23.206	21
22	0.11114	0.11135	23.110	23.105	22
23	0.11517	0.11539	23.005	23.000	23
24	0.11937	0.11959	22.896	22.891	24
25	0.12373	0.12395	22.783	22.777	25
26	0.12826	0.12848	22.665	22.659	26
27	0.13297	0.13320	22.543	22.537	27
28	0.13787	0.13809	22.415	22.410	28
29	0.14296	0.14318	22.283	22.277	29
30	0.14825	0.14847	22.146	22.140	30
31	0.15375	0.15396	22.003	21.997	31
32	0.15946	0.15966	21.854	21.849	32
33	0.16539	0.16558	21.700	21.695	33
34	0.17154	0.17172	21.540	21.535	34
35	0.17792	0.17809	21.374	21.370	35
36	0.18454	0.18471	21.202	21.198	36
37	0.19141	0.19157	21.023	21.019	37
38	0.19853	0.19868	20.838	20.834	38
39	0.20590	0.20605	20.647	20.643	39
40	0.21354	0.21369	20.448	20.444	40
41	0.22145	0.22160	20.242	20.238	41
42	0.22963	0.22978	20.030	20.026	42
43	0.23810	0.23826	19.809	19.805	43
44	0.24685	0.24702	19.582	19.578	44
45	0.25590	0.25608	19.347	19.342	45
46	0.26524	0.26543	19.104	19.099	46
47	0.27488	0.27509	18.853	18.848	47
48	0.28483	0.28506	18.594	18.589	48
49	0.29508	0.29533	18.328	18.321	49
50	0.30564	0.30591	18.053	18.046	50
51	0.31651	0.31681	17.771	17.763	51
52	0.32768	0.32801	17.480	17.472	52
53	0.33915	0.33953	17.182	17.172	53
54	0.35091	0.35135	16.876	16.865	54
55	0.36297	0.36347	16.563	16.550	55
56	0.37530	0.37589	16.242	16.227	56
57	0.38791	0.38859	15.914	15.897	57
58	0.40079	0.40158	15.580	15.559	58

Age x	$A_{[x]}$	A_x	$\ddot{a}_{[x]}$	$\ddot{a}_x$	Age x
59	0.41391	0.41483	15.238	15.214	59
60	0.42728	0.42834	14.891	14.863	60
61	0.44087	0.44209	14.537	14.506	61
62	0.45467	0.45607	14.179	14.142	62
63	0.46866	0.47025	13.815	13.774	63
64	0.48281	0.48462	13.447	13.400	64
65	0.49711	0.49916	13.075	13.022	65
66	0.51154	0.51384	12.700	12.640	66
67	0.52608	0.52863	12.322	12.256	67
68	0.54069	0.54352	11.942	11.869	68
69	0.55536	0.55847	11.561	11.480	69
70	0.57005	0.57345	11.179	11.090	70
71	0.58476	0.58843	10.796	10.701	71
72	0.59945	0.60338	10.414	10.312	72
73	0.61411	0.61827	10.033	9.925	73
74	0.62870	0.63306	9.654	9.541	74
75	0.64322	0.64772	9.276	9.159	75
76	0.65765	0.66222	8.901	8.782	76
77	0.67197	0.67653	8.529	8.410	77
78	0.68613	0.69062	8.161	8.044	78
79	0.70005	0.70445	7.799	7.684	79
80	0.71360	0.71800	7.446	7.332	80
81	0.72672	0.73123	7.105	6.988	81
82	0.73942	0.74413	6.775	6.652	82
83	0.75173	0.75668	6.455	6.326	83
84	0.76365	0.76884	6.145	6.010	84
85	0.77518	0.78060	5.845	5.704	85
86	0.78629	0.79195	5.556	5.409	86
87	0.79698	0.80287	5.279	5.125	87
88	0.80724	0.81336	5.012	4.853	88
89	0.81705	0.82340	4.757	4.592	89
90	0.82642	0.83299	4.513	4.342	90
91		0.84213		4.105	91
92		0.85081		3.879	92
93		0.85905		3.665	93
94		0.86683		3.462	94
95		0.87417		3.272	95
96		0.88106		3.092	96
97		0.88751		2.925	97
98		0.89351		2.769	98
99		0.89904		2.625	99
100		0.90404		2.495	100

End of Appendix.